CIE IGCSE Maths: Core

Your notes

Averages & Range

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Mean, Median & Mode

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Mean, Median & Mode

What is the mode?

- The **mode** is the value that appears the **most often**
  - The mode of 1, 2, 2, 5, 6 is 2
- There can be **more than one** mode
  - The modes of 1, 2, 2, 5, 5, 6 are 2 and 5
- The mode can also be called the **modal value**

What is the median?

- The **median** is the **middle** value when you put values **in size order**
  - The median of 4, 2, 3 can be found by
    - ordering the numbers: 2, 3, 4
    - and choosing the middle value, 3
- If you have an **even** number of values, find the **midpoint** of the **middle two** values
  - The median of 1, 2, 3, 4 is 2.5
    - 2.5 is the midpoint of 2 and 3
  - The **midpoint** is the **sum** of the **two middle** values **divided by 2**

What is the mean?

- The **mean** is the **sum** of the values **divided** by the **number** of values
  - The mean of 1, 2, 6 is $(1 + 2 + 6) \div 3 = 3$
- The mean can be **fraction** or a **decimal**
  - It may need **rounding**
  - You do **not** need to force it to be a **whole** number
    - You **can** have a mean of 7.5 people, for example!

How do I know when to use the mode, median or mean?

- The mode, median and mean are different ways to measure an **average**
- In certain situations it is **better** to use one average over another
- For example:
  - If the data has **extreme values** (outliers) like 1, 1, 4, 50
    The mode is 1
    The median is 2.5
    The mean is 14
    - **Don't use the mean** (it's badly affected by extreme values)
  - If the data has **more** than one mode
    - **Don't use the mode** as it is not clear
  - If the data is **non-numerical**, like dog, cat, cat, fish
    - You can **only** use the **mode**

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Worked example

15 students were timed to see how long it took them to solve a mathematical problem. Their times, in seconds, are given below.

12	10	15	14	17
11	12	13	9	21
14	20	19	16	23

(a) Find the mean time, giving your answer to 3 significant figures.

Add up all the numbers (you can add the rows if it helps)

$12 + 10 + 15 + 14 + 17 = 68$
$11 + 12 + 13 + 9 + 21 = 66$
$14 + 20 + 19 + 16 + 23 = 92$

$\text{Total} = 68 + 66 + 92 = 226$

Divide the total by the number of values (there are 15 values)

$$\frac{226}{15} = 15.066\,666\,\ldots$$

Write the mean to 3 significant figures
Remember to include the units

The mean time is 15.1 seconds (to 3 s.f.)

(b) Find the median time.

Write the times in order and find the middle value

~~9~~ ~~10~~ ~~11~~ ~~12~~ ~~12~~ ~~13~~ ~~14~~ (14) ~~15~~ ~~16~~ ~~17~~ ~~19~~ ~~20~~ ~~21~~ ~~23~~

The median time is 14 seconds

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(c) Explain why the median is a better measure of average time than the mode.

Try to find the mode (the number that occurs the most)

There are two modes: 12 and 14

Explain why the median is better

There is no clear mode (there are two modes, 12 and 14), so the median is better

(d) If a 16th student has a time of 95 seconds, explain why the median of all 16 students would be a better measure of average time than the mean.

The16th value of 95 is extreme (very high) compared to the other values
Means are affected by extreme values

The mean will be affected by the extreme value of 95 whereas the median will not


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Calculations with the Mean

How do I solve harder problems involving the mean?


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- Remember what the mean is
  - **Mean = total of values ÷ number of values**
    - It is a **formula** involving three quantities
    - if you know any two, you can find the other one
- A question may require you to work **backwards** from a **known mean**
  - It helps to rearrange the formula
  - **Total of values = mean × number of values**
- Find the **total** of the values **before and after** to help with question that involve:
  - **missing** values
  - **adding in**, or **taking out**, a value

Examiner Tip

- It helps to start thinking of the mean as a formula which you can rearrange
  - Total of values = mean × number of values

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Worked example

A class of 24 students has a mean height of 1.56 metres.
A new student joins the class.
The mean height of the class is now 1.57 metres.

Find the height of the new student.

Rearrange the formula for mean to get 'total of heights = mean height × number of students'
Find the total of heights before

$$\text{Total of heights before} = 1.56 \times 24 = 37.44$$

Find the total of heights after
Remember there are now 25 students

$$\text{Total of heights after} = 1.57 \times 25 = 39.25$$

The height of the new student is the difference of the two totals above

$$39.25 - 37.44 = 1.81$$

The height of the new student is 1.81 metres

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Range

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Range

What is the range?

- The **range** is the **difference** between the **highest value** and the **lowest value**
  - range = highest - lowest
    - For example, the range of 1, 2, 5, 8 is 8 - 1 = 7
- It measures how **spread out** the data is
  - Ranges of different data sets can be **compared** to see which is more spread out
- Be careful with **negatives**
  - The range of -2, -1, 0, 4 is 4 - (-2) = 6

How do I know when to use the range?

- The **range** is a simple measure of how **spread out** the data is
  - The range does **not** measure an average value
- It should **not** be used if there are any **extreme values** (outliers)
  - For example, the range of 1, 2, 5, 80 is 80 - 1 = 79
    - This is not a good measure of spread
    - The range is affected by extreme values

Examiner Tip

- If asked to find the range in an exam, make sure you show your subtraction clearly (don't just write down the answer).

Worked example

Find the range of the following data.

3.4	4.2	2.8	3.6	9.2	3.1	2.9	3.4	3.2
3.5	3.7	3.6	3.2	3.1	2.9	4.1	3.6	3.8
3.4	3.2	4.0	3.7	3.6	2.8	3.9	3.1	3.0

Range = highest value - lowest value

9.2 - 2.8

The range is 6.4

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Averages from Tables

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Averages from Tables

What are frequency tables?

- **Frequency tables** are used to **summarise** data in a **neat** format
  - They also put the data in **order**
- For example, the table below shows the number of pets in different houses along a street
  - The number of pets is the **data value**, ***x***
  - The number of houses is the **frequency**, ***f***
    - The **frequency** is **how many times** a data value is **recorded** (or **seen**)
- The **total frequency**, ***n***, can be calculated by **adding** together all the values in the **frequency column**

Number of pets (data *value*, *x*)	Number of houses (*frequency*, *f*)
0	2
1	7
2	6
3	4
4	1
Total frequency (n) = 20	

How do I find the mode from a frequency table?

- The **mode** is the **data value** with the **highest frequency**
  - The mode for the example above is 1 pet per house
    - The mode is **not** the frequency, 7, this is the number of houses that have exactly 1 pet

How do I find the median from a frequency table?

- The **median** is the **data value** in the **middle** of the **frequency**
  - It is the $\left(\frac{n+1}{2}\right)^{th}$ value, where n is the total frequency
- From above, $n = 20$ so the median is the $\left(\frac{20+1}{2}\right)^{th}$ = 10.5^{th} value in the table
  - The first two rows have a **combined** (**cumulative**) frequency of 2 + 7 = 9
  - The first three rows have a combined frequency of 2 + 7 + 6 = 15
  - Therefore the 10th and 11th values are in the third row (x = 2)
    - The median is 2 pets per house

How do I find the mean from a frequency table?

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- The **mean** from a **frequency table** has the following **formula**:
  - $\text{mean} = \dfrac{\text{total of 'data value} \times \text{frequency'}}{\text{total frequency}}$
    - It helps to create a new column of 'data value × frequency'
    - Add up the values in this column
    - Divide by the total frequency
- The mean is $\dfrac{35}{20}$ = 1.75 pets per house
  - Means do not need to be whole numbers

Number of pets (data *value*, *x*)	Number of houses (frequency, *f*)	data value × frequency (*xf*)
0	2	0 × 2 = 0
1	7	1 × 7 = 7
2	6	2 × 6 = 12
3	4	3 × 4 = 12
4	1	4 × 1 = 4
	Total = 20	Total = 35

How do I find the range from frequency tables?

- The **range** is the **difference** of the largest and smallest **data values**
  - The range above is 4 - 0 = 4
    - The range is **not** the difference of the largest and smallest frequencies

What else should I know about frequency tables?

- **Tables** can be **converted** back into a **list of data values** using their **frequencies**
  - From above, 0 pets were recorded twice, 1 pet was recorded 7 times, 2 pets were recorded 6 times, etc
    - The list of pets recorded is 0,0,1,1,1,1,1,1,1,1,2,2,2,2,2,2,3,3,3,3,4
- You could then find the **mode**, **median** and **mean** from this list of numbers

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Worked example

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The table shows data for the shoe sizes of pupils in class 11A.

Shoe size	Frequency
6	1
6.5	1
7	3
7.5	2
8	4
9	6
10	11
11	2
12	1

(a) Find the mean shoe size for the class, giving your answer to 3 significant figures.

It helps to label shoe size (x) and frequency (f)
Add an extra column and calculate the values of 'shoe size × frequency', (xf)
Find the total frequency and total xf value

Shoe size (x)	Frequency (f)	xf
6	1	$6 \times 1 = 6$
6.5	1	$6.5 \times 1 = 6.5$
7	3	$7 \times 3 = 21$
7.5	2	$7.5 \times 2 = 15$
8	4	$8 \times 4 = 32$
9	6	$9 \times 6 = 54$
10	11	$10 \times 11 = 110$
11	2	$11 \times 2 = 22$
12	1	$12 \times 1 = 12$
	Total = 31	Total = 278.5

Use the formula that the mean is the total of the xf column divided by the total frequency

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$$\text{Mean} = \frac{278.5}{31} = 8.983\ 870\ ...$$

Give your final answer to 3 significant figures

The mean shoe size is 8.98 (to 3 s.f.)

Note that the mean does not have to be an actual shoe size

(b) Find the median shoe size.

The median is the $\left(\frac{n+1}{2}\right)^{th}$ value where n is the total frequency

$$\frac{n+1}{2} = \frac{31+1}{2} = \frac{32}{2} = 16$$

The median is the 16^{th} value
There are 1 + 1 + 3 + 2 + 4 = 11 values in the first five rows of the table
There are 11 + 6 = 17 values in the first six rows of the table
Therefore the 16^{th} value must be in the sixth row

The median shoe size is 9

(c) Find the range of the shoe sizes.

The range is the highest shoe size subtract the lowest show size

12 - 6

The range of the shoe sizes is 6

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Comparing Distributions

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Comparing Data Sets

How do I compare two data sets?

- You may be given **two** sets of data that relate to a context
- To **compare** data sets, you need to
  - compare their **averages**
    - Mode, median or mean
  - compare their **spreads**
    - Range

How do I write a conclusion when comparing two data sets?

- When comparing **averages** and **spreads**, you need to
  - compare **numbers**
  - **describe** what this means in **real life**
- **Copy** the **exact** wording from the question in your answer
- There should be **four** parts to your conclusion
  - For example:
    - "The **median** score of class A (45) is **higher** than the median score of class B (32)."
    - "This means class A **performed better** than class B in the test."
    - "The **range** of class A (5) is lower than the range of class B (12)."
    - "This means the scores in class A were **less spread out** than scores in class B."
  - Other good phrases for lower ranges include:
    - "scores are **closer together**"
    - "scores are **more consistent**"
    - there is **less variation** in the scores"

What restrictions are there when drawing conclusions?

- The data set may be **too small** to be truly representative
  - Measuring the heights of only 5 pupils in a whole school is not enough to talk about averages and spreads
- The data set may be **biased**
  - Measuring the heights of just the older year groups in a school will make the average appear too high
- The conclusions might be influenced by **who** is presenting them
  - A politician might choose to compare a different type of average if it helps to strengthen their argument!

What else could I be asked?

- You may need to **choose** which, out of mode, median and mean, to compare
  - Check for **extreme values** (outliers) in the data

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- Avoid using the **mean** as it is affected by extreme values
- You may need to think from the **point of view** of another person
  - A teacher might not want a large spread of marks
    - It might show that they haven't taught the topic very well!
  - An examiner might want a large spread of marks
    - It makes it clearer when assigning grade boundaries, A, B, C, D, E, ...


Your notes

Examiner Tip

- When comparing data sets in the exam, half the marks are for comparing the numbers and the other half are for saying what this means in real life.

Worked example

Julie collects data showing the distances travelled by snails and slugs during a ten-minute interval. She records a summary of her findings, as shown in the table below.

	Median	Range
Snails	7.1 cm	3.1 cm
Slugs	9.7 cm	4.5 cm

Compare the distances travelled by snails and slugs during the ten-minute interval.

Compare the numerical values of the median (an average)
Describe what this means in real life

Slugs have a higher median than snails (9.7 cm > 7.1 cm)
This suggests that, on average, slugs travel further than snails

Compare the numerical values of the range (the spread)
Describe what this means in real life

Snails have a lower range than slugs (3.1 cm < 4.5 cm)
This suggests that there is less variation in the distances travelled by snails

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