OCR A Level Maths: Mechanics

Your notes

2.4 Variable Acceleration – 2D

Contents

2.4.1 Using Calculus in 2D

Your notes

Using Calculus in 2D

How can I use vectors in 2D Kinematics?

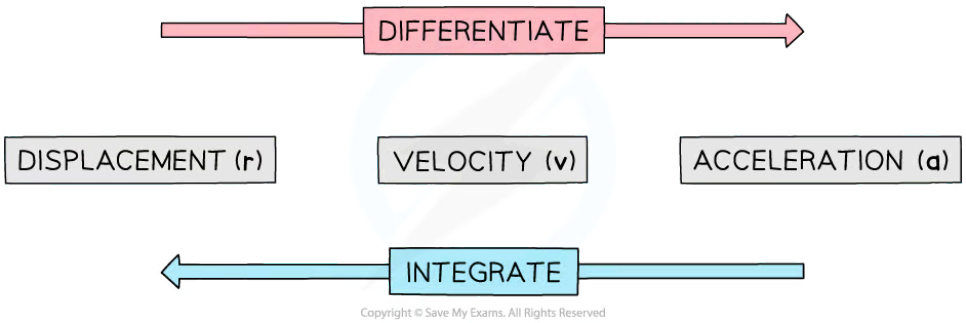
- It is important you understand **Calculus in 1D** first
- Also recall the differences between key **vector** and **scalar** quantities
- **Displacement** - from starting point
- **Distance** – from start/origin/another particle (all could be different)
- **Speed** and **Velocity**
- **acceleration** and **magnitude** of **acceleration/deceleration**
- **time** is a scalar quantity

How do I use calculus with vectors in 2D?

- To **differentiate** a vector:
  - Differentiate both (**i** and **j**) components of the vector
- **To integrate** a vector:
  - Integrate both (**i** and **j**) components of the vector
  - There will be a constant of integration, **c**, which will be a vector made up of two values (an **i**-component constant and a **j**-component constant)
  - Find **c** by substituting in any known vectors, in the same way as you would if it were in 1D
    - Find the separate components of the vector **c** by equating the **i**-components and the **j**-components separately

How does calculus link to kinematics?

- In 2D we normally use the vector **r** to represent displacement instead of s
- Just like in 1D:
  - **Differentiate** displacement to get velocity and velocity to get acceleration
  - **Integrate** acceleration to get velocity and velocity to get displacement



Harder problems involving vectors, calculus, position vectors and F=ma

- Harder problems could involve:
  - using Newton's Second Law (N2L) **F** = *m***a**
  - two vectors being equal requiring both **i** and **j** components to be equal
  - exponentials, logarithms, trigonometric functions (not just polynomials)
- suvat equations (in vector form) would only be involved if **acceleration is constant**
- You may have to find the **magnitude** of the vectors
  - The magnitude of the **velocity** is the **speed**
  - The magnitude of the **displacement** is the **distance** from the **starting point**
- The position vector of a particle is $\mathbf{r} = \mathbf{r_0} + \mathbf{s}$
  - $\mathbf{r_0}$ is the **initial position** of the particle
  - **s** is the **displacement** of the particle from its initial position

Your notes

Worked example

A ship moves with velocity, $\mathbf{v}$ m s^{-1}, where $\mathbf{v} = (2t)\mathbf{i} + (e^{0.5t})\mathbf{j}$. Initially the ship has position vector $(3\mathbf{i} - 4\mathbf{j})$ m from a fixed origin O.

(a) Find an expression for the position vector of the ship after t seconds.

(b) Find the magnitude of the acceleration when $t = 3$.

(a) Find an expression for the position vector of the ship after t seconds.

Your notes

a) Step 1: Identify whether to differentiate or integrate

Displacement is the integral of velocity

Step 2: Integrate velocity

$\mathbf{r} = \int \mathbf{v}\, dt$

$\mathbf{r} = \int (2t)\mathbf{i} + (e^{0.5t})\mathbf{j}\, dt$

$\mathbf{r} = (t^2)\mathbf{i} + (2e^{0.5t})\mathbf{j} + \mathbf{c}$

Remember constant of integration is a vector

Step 3: Find "+c"

$t = 0 \quad \mathbf{r} = 3\mathbf{i} - 4\mathbf{j}$

$3\mathbf{i} - 4\mathbf{j} = 0\mathbf{i} + 2\mathbf{j} + \mathbf{c}$

$\mathbf{c} = 3\mathbf{i} - 4\mathbf{j} - 2\mathbf{j}$

$\mathbf{c} = 3\mathbf{i} - 6\mathbf{j}$

$\mathbf{r} = ((t^2 + 3)\mathbf{i} + (2e^{0.5t} - 6)\mathbf{j})\text{m}$

(b) Find the magnitude of the acceleration when $t = 3$.

b) Step 1: Identify whether to differentiate or integrate

Acceleration is the derivative of velocity

Step 2: Differentiate velocity

$\mathbf{a} = \frac{d\mathbf{v}}{dt}$

$\mathbf{a} = \frac{d}{dt}((2t)\mathbf{i} + (e^{0.5t})\mathbf{j})$

$\mathbf{a} = 2\mathbf{i} + (0.5e^{0.5t})\mathbf{j}$

Step 3: Find acceleration at the stated time

$t = 3 \quad \mathbf{a} = 2\mathbf{i} + 0.5e^{1.5}\mathbf{j}$

Step 4: Find the magnitude

$|\mathbf{a}| = \sqrt{2^2 + (0.5e^{1.5})^2}$

$|\mathbf{a}| = 3.0035...$

$|\mathbf{a}| = 3.00\ \text{ms}^{-2}$ (3 sf)

Examiner Tip

- If the question refers to the direction that the particle is travelling, then you would use the velocity. If the **direction of motion** is asked for then it should be clear from the question whether they want an angle, bearing or vector which it is parallel to.


Your notes