Cambridge (CIE) IGCSE Maths: Extended

Your notes

Circle Theorems

Contents

- Angles at Centre & Circumference
- Angle in a Semicircle
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- Angles in Cyclic Quadrilaterals
- Angles in the Same Segment
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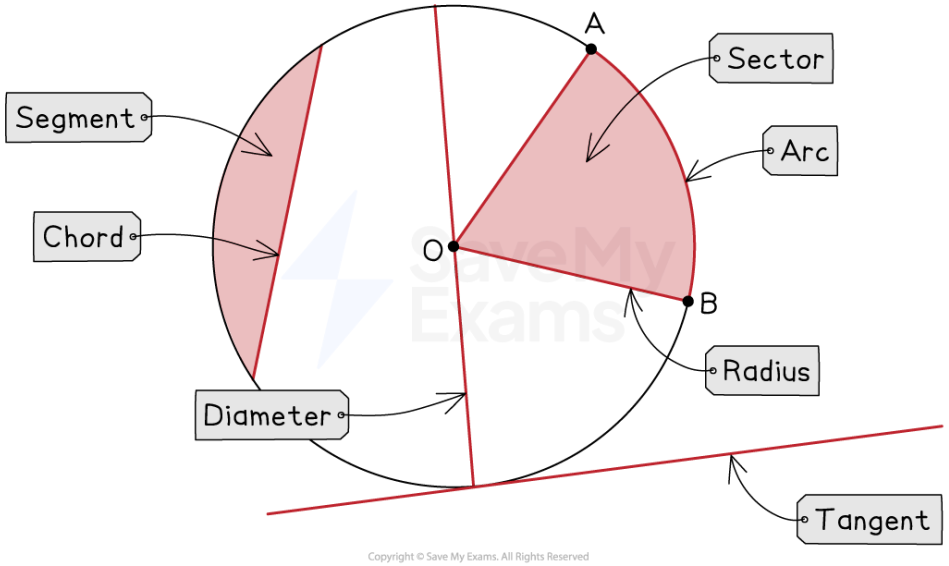
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Your notes

Angles at centre & circumference

What are circle theorems?

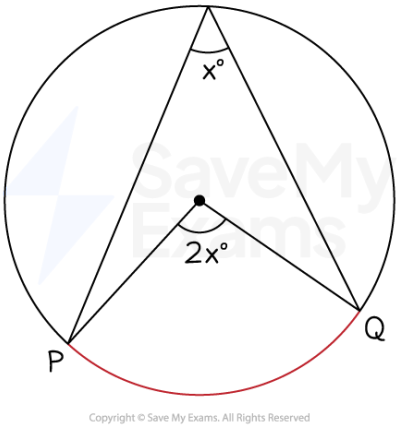
- **Circle Theorems** deal with angles that occur when lines are drawn within (and connected to) a circle
- You may need to use other **facts and rules** such as:
  - basic properties of lines and angles
  - properties of triangles and quadrilaterals
  - angles in parallel lines or polygons



Parts of a circle

Circle Theorem: The angle at the centre is twice the angle at the circumference

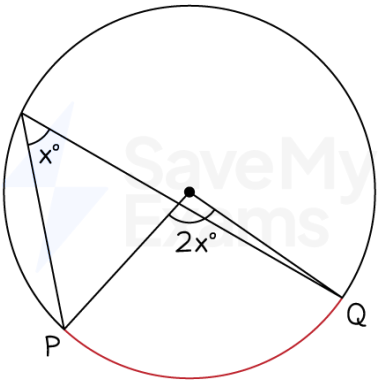
- In this theorem, the **chords (radii) to the centre** and the **chords to the circumference** are both drawn from (subtended by) the ends of the **same arc**



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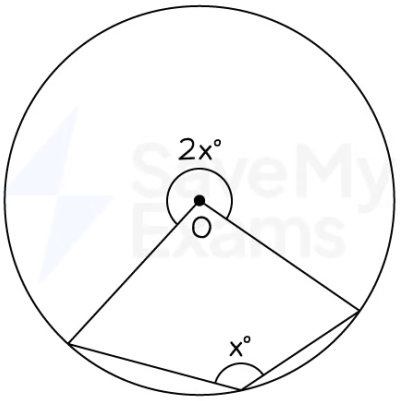
Your notes

- To spot this circle theorem on a diagram
  - find any **two radii** in the circle and follow them to the **circumference**
  - see if there are **lines from those points** going to **any other point** on the **circumference**
  - it may look like the shape of an **arrowhead**
- When explaining this theorem in an exam you must use the keywords:
  - **The angle at the centre is twice the angle at the circumference**
- This theorem is still true when the **'triangle parts' overlap**



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- It is also true when the lines form a **diamond shape**
  - You need to compare the **reflex angle at the centre** with the angle at the circumference
  - **Common mistakes** are to
    - compare the wrong angles
    - confuse it with a different circle theorem on cyclic quadrilaterals



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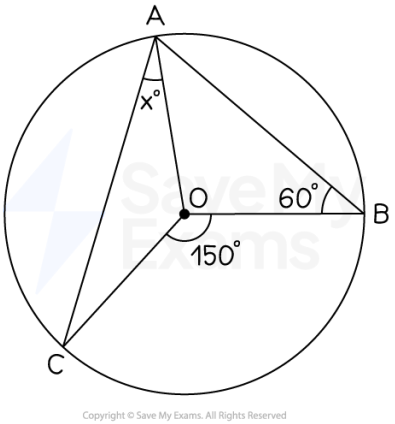
Examiner Tips and Tricks

Questions often say to give "reasons" for your answer.

Quote an **angle fact** or **circle theorem** for **every** angle you find (not just one for the final answer).

Worked Example

Find the value of x in the diagram below.



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Not drawn accurately

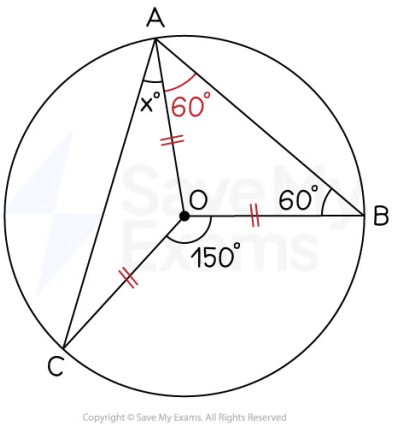
Give a reason for each step of your working.

Answer:

There are three radii in the diagram, *AO*, *BO* and *CO*
Mark these as equal length lines

Notice how they create two isosceles triangles
Base angles in isosceles triangles are equal

Angle OAB = angle OBA = 60° (isosceles triangle)



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Use the circle theorem:

The angle at the centre is twice the angle at the circumference

Form an equation for x

$$2(x + 60) = 150$$

Expand the brackets and solve the equation

$$2x + 120 = 150$$
$$2x = 30$$
$$x = 15$$

$$x = 15$$

Base angles in isosceles triangles are equal

The angle at the centre is twice the angle at the circumference

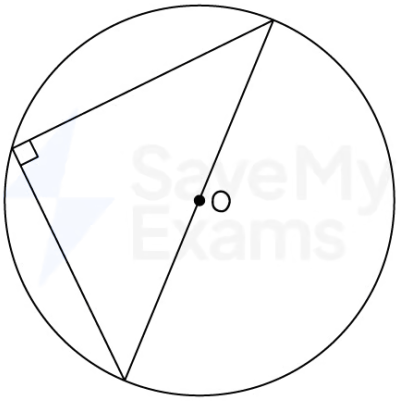
Your notes

Your notes

Angle in a semicircle

Circle Theorem: The angle in a semicircle is 90°

- The lines drawn from a **point** on the **circumference** to either end of a **diameter** are **perpendicular**
  - The angle at that point on the circumference is **90°**
  - This circle theorem only uses half of the circle
    - The right-angle is called the angle in a **semicircle**
- This is a **special case** of the circle theorem "the angle at the centre is twice the angle at the circumference"
  - The angle on the diameter is 180°
  - The angle at the circumference is halved, giving 90°



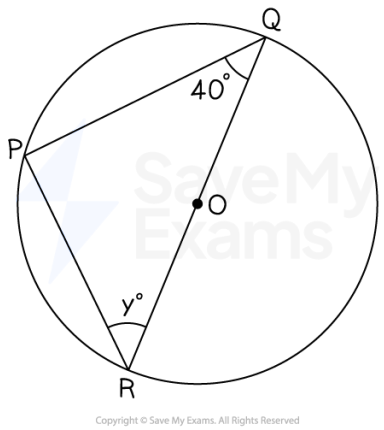
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- To spot this circle theorem on a diagram look for a triangle where
  - **one side** is the **diameter**
    - Remember that a diameter always goes through the **centre**
  - **all three vertices** are on the **circumference**
- The 90° angle will always be the angle **opposite** the diameter
- When explaining this theorem in an exam you must use the **keywords:**
  - **The angle in a semicircle is 90°**
- Questions that use this theorem may
  - appear in **whole circles** or in **semicircles**

- require the use of **Pythagoras' Theorem** to find a missing length

Your notes

Worked Example



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P, *Q* and *R* are points on a circle.
RQ is a diameter.

Find the value of y.

Give a reason for your answer.

Answer:

Use the fact that angles in a triangle add up to 180° and the circle theorem

The angle in a semicircle is 90°

Write an equation for y

$$y + 90 + 40 = 180$$

Solve for y

$$y = 180 - 90 - 40$$
$$y = 50$$

$$y = 50$$

The angle in a semicircle is 90°

Your notes

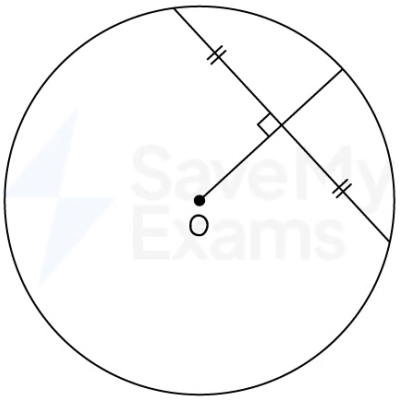
Circles & chords

What is a chord?

- A **chord** is any straight line in a circle that joins any **two points** on the **circumference**
  - Chords of **equal length** are **equidistant** (the same distance) from the centre

Circle Theorem: The perpendicular bisector of a chord passes through the centre

- If a **line through the centre** (such as a **radius or diameter**) goes through the **midpoint** of a **chord**
  - it will **bisect** (cut in half) that chord at **right angles** to it



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- To spot this circle theorem on a diagram
  - look for a **radius** and see if it **intersects** any **chords**
  - or look to see if you could **draw** a radius that bisects a chord
- When explaining this theorem in an exam you can use either phrase below:
  - **A radius bisects a chord at right angles**
  - **The perpendicular bisector of a chord passes through the centre**

Examiner Tips and Tricks

Look out for **isosceles triangles** formed by a chord and two radii.

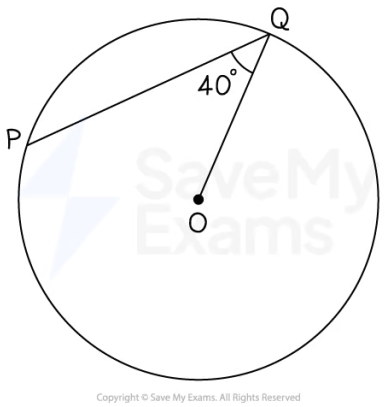
Two angles in the triangle will be equal and there will be at least one line of symmetry.

Your notes

Worked Example

The diagram below shows a circle with centre, O.
Two points, P and Q, lie on its circumference.
The radius of the circle is 6 cm.
Angle $OQP = 40°$.

Find the length PQ.

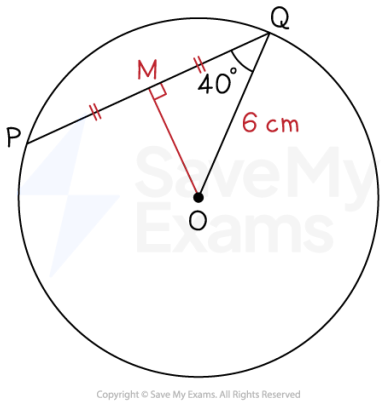


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Answer:

Label the radius on the diagram as 6 cm

Draw a line from O to the midpoint, M, of the line PQ
The angle formed between OM and PQ will be a right angle



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Use SOHCAHTOA on triangle OMQ to find the length MQ

$$\cos 40 = \frac{MQ}{6}$$

$$6 \cos 40 = MQ$$

$$MQ = 4.59626...$$

Double MQ to find the length PQ

$$4.59626... \times 2 = 9.19253...$$

Round to 3 significant figures

$PQ = 9.19$ cm (3 s.f.)

Your notes

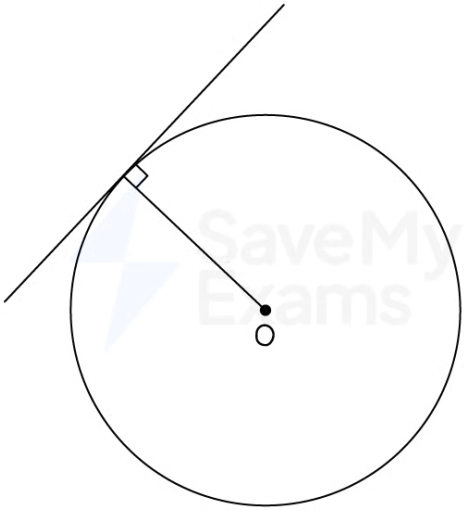
Circles & tangents

What is a tangent?

- A tangent to a circle is a straight line **outside of the circle** that touches its **circumference** at exactly **one point**

Circle Theorem: A radius and a tangent meet at right angles

- If a **radius and a tangent** meet at a point on the **circumference** of a circle, the **angle** formed between them will be **90°**
  - They are **perpendicular** to each other

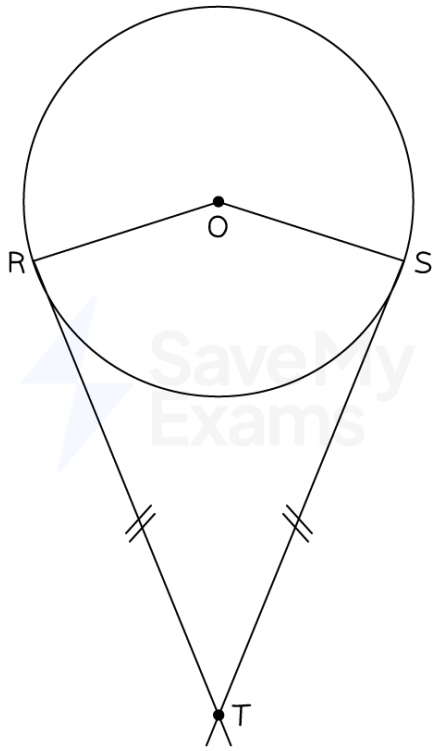


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- When explaining this theorem in an exam you must use the keywords:
  - **A radius and a tangent meet at right angles**

Circle Theorem: Tangents from an external point are equal in length

- **Two tangents** from the **same external point** are equal in length
- This means that a **kite** can be formed by two tangents meeting a circle
  - The kite below has a vertical line of symmetry
    - It is formed from two **congruent** triangles back-to-back
  - The kite will have **two right angles** where the tangents meet the radii
    - You can use **Pythagoras** and **SOHCAHTOA** on each of these triangles



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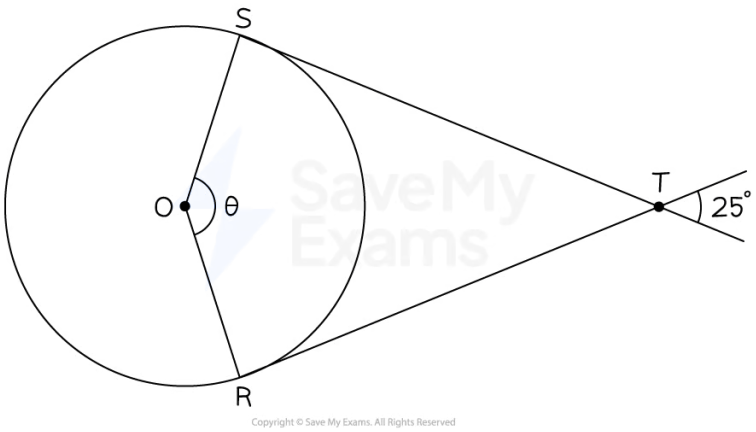
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Examiner Tips and Tricks

Look for tangents in the exam and draw on the radius at right angles to see if it helps.

Worked Example

Find the value of θ in the diagram below.



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Answer:

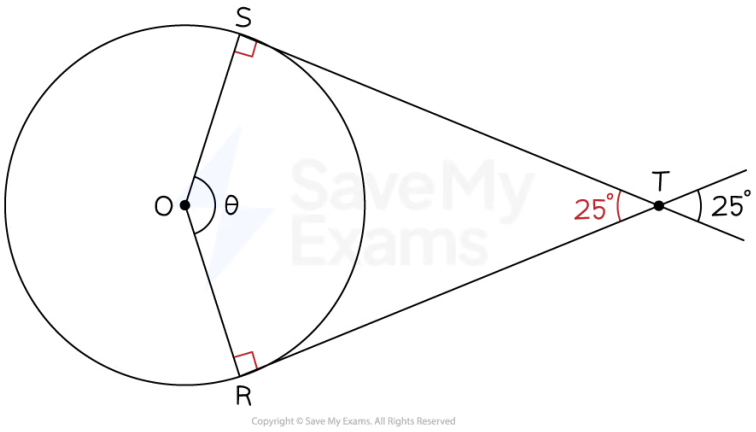
The lines *ST* and *RT* are both tangents to the circle
They meet the two radii on the circumference at the points *S* and *R*

Angle *TSO* = angle *TRO* = 90°
A radius and a tangent meet at right angles

Use vertically opposite angles to find the value of angle *RTS*

Angle *RTS* = 25°
Vertically opposite angles

Mark these angles clearly on the diagram



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Angles in a quadrilateral add up to 360°
Use this to form an equation for θ

$$\theta + 90 + 90 + 25 = 360$$

Angles in a quadrilateral sum to 360°

Simplify

$$\theta + 205 = 360$$

Solve

$$\theta = 360 - 205$$

$$\theta = 155°$$



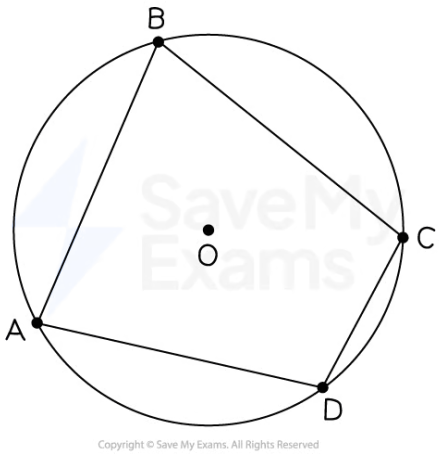
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Cyclic quadrilaterals

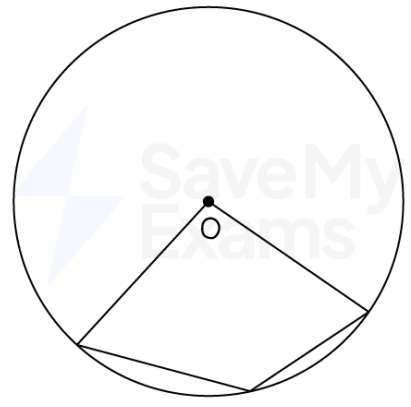
Circle theorem: Opposite angles in a cyclic quadrilateral add up to 180°

- A quadrilateral that is formed by **four points** on the **circumference** of a circle, (a **cyclic quadrilateral**), will have pairs of **opposite angles** that add up to **180°**



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- To spot this theorem in a diagram
  - look for **quadrilaterals** that have all four points on the **circumference**
- When explaining this theorem in an exam you must use the keywords:
  - **Opposite angles in a cyclic quadrilateral add up to 180°**
- The theorem only works for **cyclic quadrilaterals**
  - The diagram below shows a common scenario that is **not** a cyclic quadrilateral



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Examiner Tips and Tricks

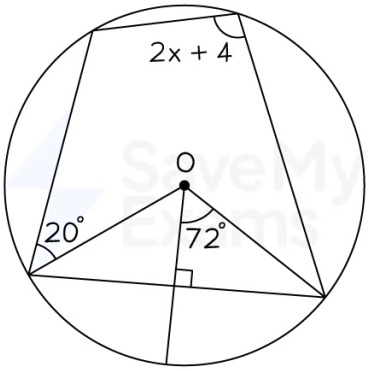
Cyclic quadrilaterals are often easy to spot in a busy diagram.

Mark on their angles (even if you think you don't need them) as they may help you later on!

Worked Example

The circle below has centre, O.

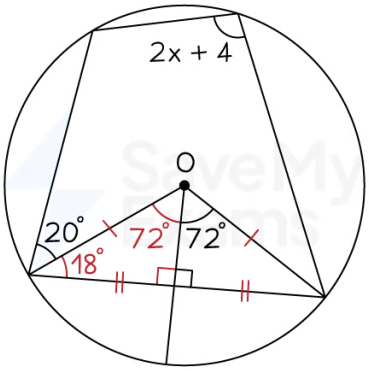
Find the value of x.



Answer:

Identify both the cyclic quadrilateral and the radius perpendicular to the chord

Add to the diagram as you work through the problem



The radius bisects the chord and so creates two congruent triangles

Use this to work out 72° (equal to the equivalent angle in the other triangle)
And 18° (angles in a triangle add up to 180°)

Then use that **opposite angles in a cyclic quadrilateral add up to 180°**

$$2x + 4 + 20 + 18 = 180$$
$$2x = 138$$

$$x = 69°$$



Your notes

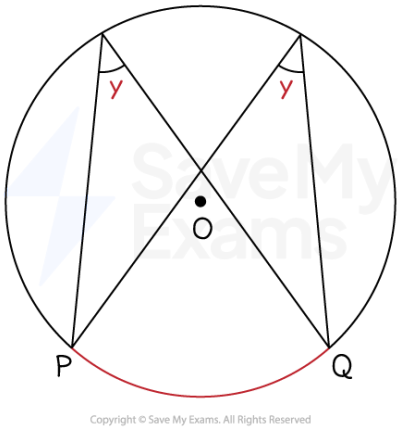
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Circles & segments

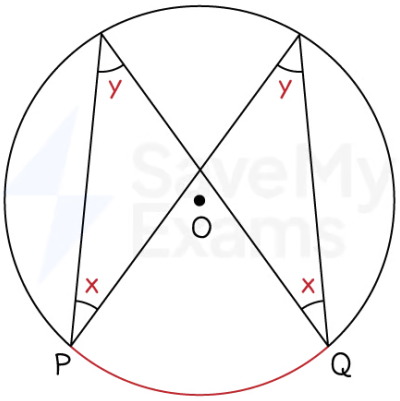
Circle Theorem: Angles in the same segment are equal

- **Any two angles** on the **circumference** of a circle that are formed from the **same two points** on the circumference are **equal**
  - These two angles are in the **same segment** of the circle
    - To see this, add the chord *PQ* below to split the circle into two segments



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- To spot this circle theorem on a diagram
  - Find **two points** on the **circumference** that meet at a **third point**
  - See if there are **any other pairs of lines** from the **same two original points** that meet at a different point on the circumference
- When explaining this theorem in an exam you must use the keywords:
  - **Angles in the same segment are equal**
- Look out for a **bowtie shape**
  - The theorem works **upside down**, in that the angles at *P* and *Q* are **also equal**



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Examiner Tips and Tricks

An exam question diagram may have **multiple equal angles**.

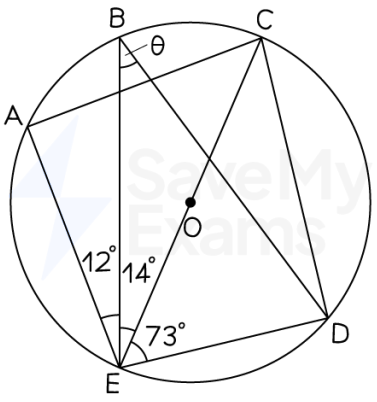
Look for as many as possible by seeing how many pairs of lines start from the same two points on the circumference.

Worked Example

The diagram below shows a circle with centre, O.
A, B, C, D and E are five points on the circumference on the circle.

Angle $AEB = 12°$.
Angle $BEC = 14°$.
Angle $CED = 73°$.
Angle $EBD = \theta°$.

Find the value of θ.



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Answer:

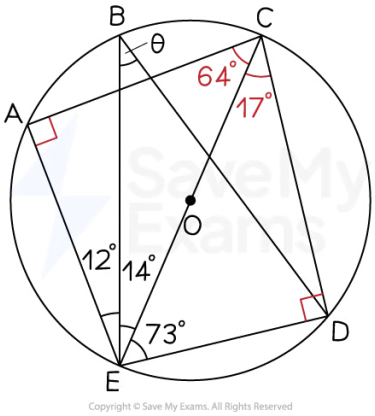
CE is a diameter
This means that triangles *EAC* and *CDE* are both triangles in a semicircle

Angle *EAC* = 90°
Angle *CDE* = 90°
Angle in a semicircle = 90°

Find the other angles in the triangles

Angle *ECA* = 64°
Angle *ECD* = 17°
Angles in a triangle = 180°

Label these angles on the diagram



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Angle θ is formed by two lines coming from either end of the chord *ED*
Angle *ECD* is also formed by two lines coming from either end of the chord *ED*

Angle θ = angle ECD = 17°
Angles in the same segment are equal

Angle θ = 17°

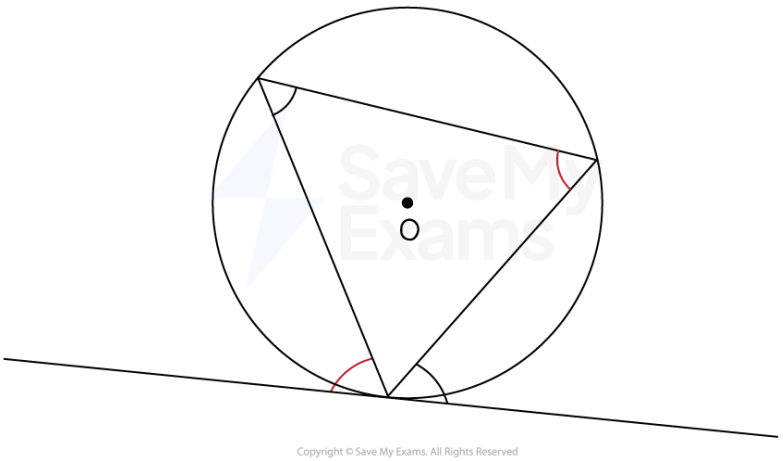
Your notes

Your notes

Alternate segment theorem

Circle theorem: The alternate segment theorem

- The **angle** between a **chord** and a **tangent** is **equal** to the **angle** in the **alternate segment**
  - The alternate segment is the region on the **opposite side** of a chord from a given angle formed between that chord and a tangent line



Example of angles in alternate segments

- To spot this circle theorem on a diagram
  - look for a **cyclic triangle**
    - where all three vertices of the triangle lie on the circumference
  - **one vertex** of the triangle meets a **tangent**
- To identify which angles are equal
  - mark the **angle** between the **tangent** and the **side** of the **cyclic triangle**
  - the angle **inside** the triangle at the corner **opposite the side** of the triangle that forms the first angle is the **equal angle**
- When explaining this theorem in an exam you can just say the phrase:
  - **The Alternate segment theorem**

Examiner Tips and Tricks

Look for **cyclic triangles** and **tangents** in busy diagrams.

Questions involving the alternate segment theorem frequently appear in exams!

Your notes

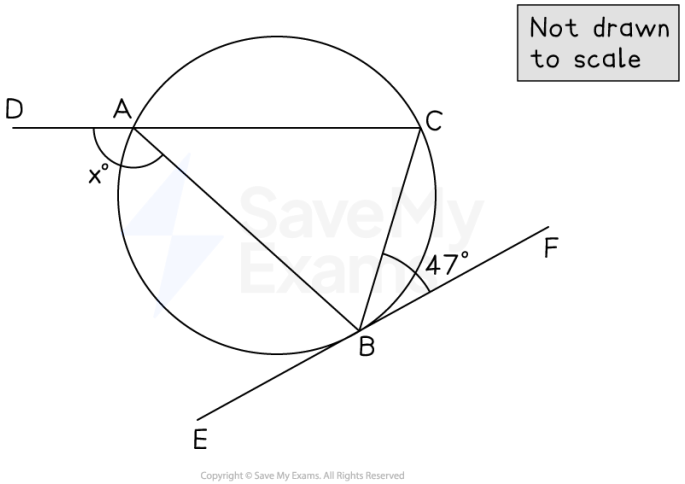
Worked Example

A, B and C are points on a circle.

DAC is a straight line.

EBF is a tangent to the circle.

Find the value of x.

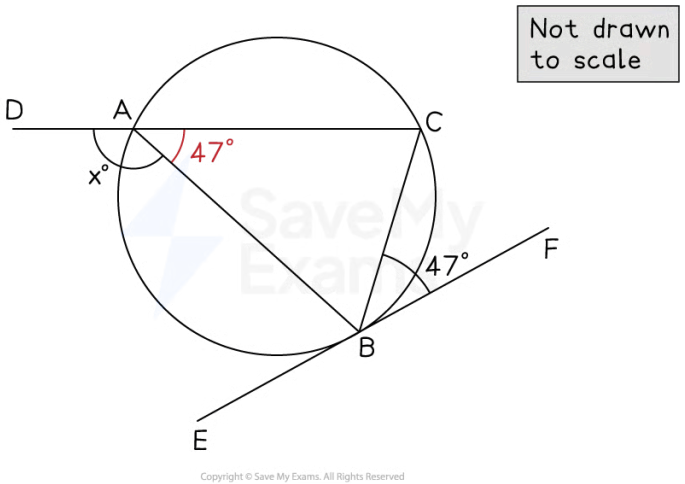


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Answer:

One vertex of this triangle meets a tangent at point *B*
The angle between one of its sides (*BC*) and the tangent is given
Find the angle inside the triangle, opposite to the same side (*BC*)

Angle *CBF* = Angle *CAB* by the alternate segment theorem



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Angle x and angle CAB form a straight lie

$$x + 47 = 180$$
$$x = 180 - 47$$
$$x = 133$$



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