OCR Gateway GCSE Physics

Your notes

2.5 Forces & Elasticity

Contents

2.5.1 Changing Shape

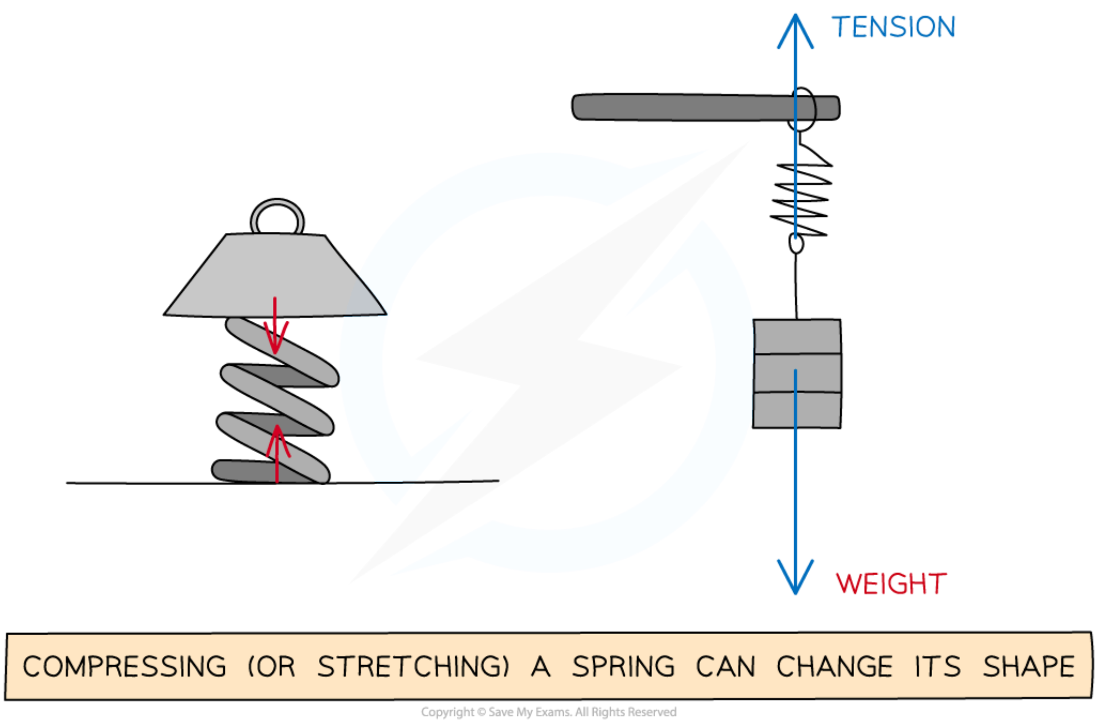
Your notes

Forces & Changing Shape

- For stationary objects, more than one force has to be applied to change their shape
- Their shape can change by:
  - **Stretching** (forces in opposite directions away from the object)
  - **Bending** (forces that deform the object)
  - **Compressing** (forces in opposite directions towards the object)
- A combination of all three shape changes can also occur

Compression

- An example of **compression** is placing a mass on top of a spring resting on a flat surface
- The two forces are:
  - The weight of the mass
  - The reaction force from the surface to the spring
- These two forces act **towards** each other



The compression or stretching of a spring requires two forces

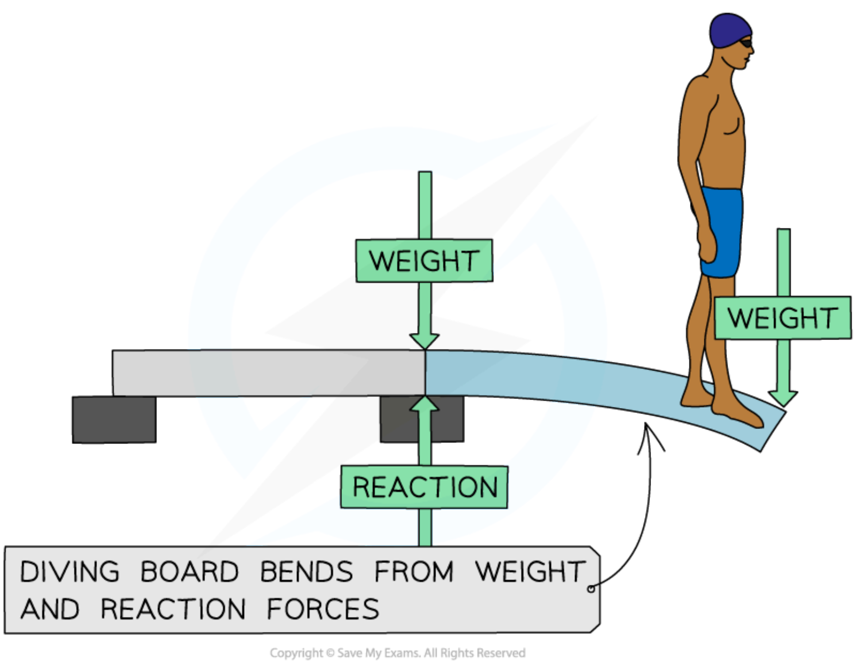
Your notes

Stretching

- An example of **stretching** is placing a mass on the bottom of a vertically hanging spring
- The two forces are:
  - The weight of the mass
  - The tension in the spring
- These two forces act **away** from each other
  - These opposite forces are a result of Newton's Third Law

Bending

- An example of **bending** is a diving board bending when a swimmer stands at the far end
- The two forces are:
  - The weight of the swimmer
  - The reaction force from the block to the dividing board
- These two forces act towards each other, but at different points on the object
- Bending can also be caused by two forces at an **angle** to each other



Forces on a diving board cause it to be bend when a swimmer stands on one end

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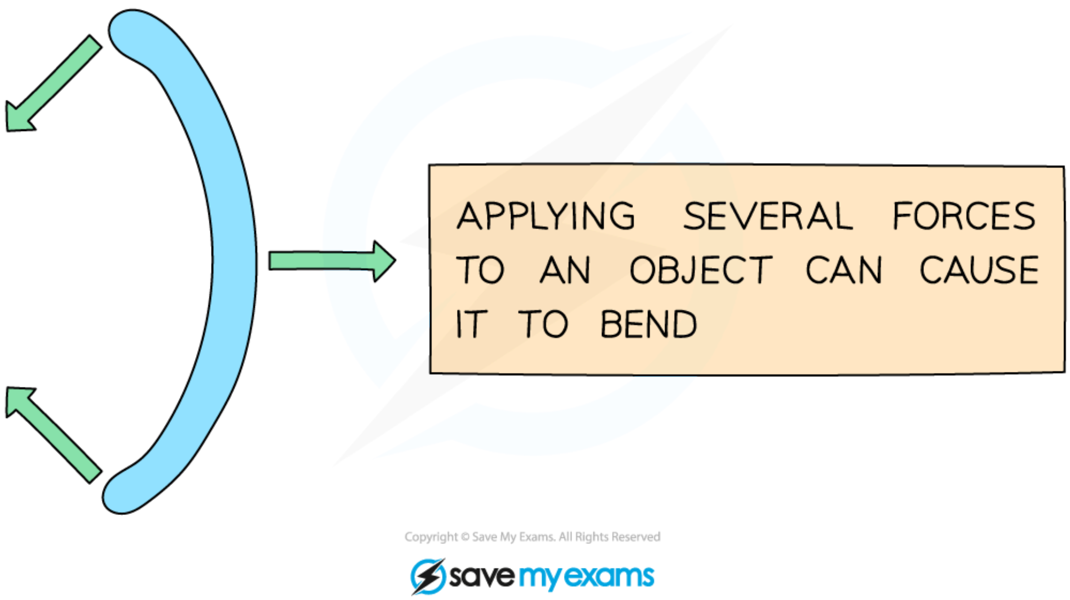
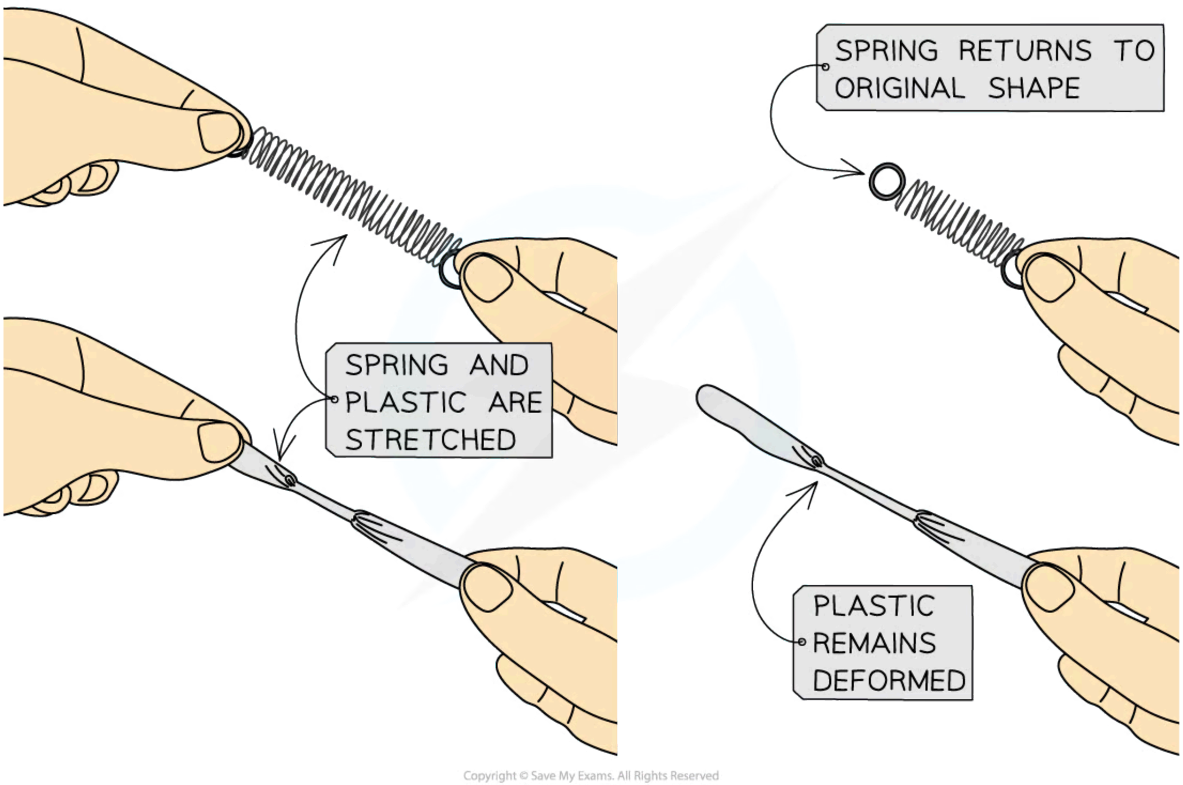


More than one force on an object can cause it to bend

Elastic & Plastic Deformation

Your notes

- When some objects, such as springs or rubber bands, are stretched they will return to their original shape and length once the forces are removed
  - Other materials, such as plastic, remain permanently deformed (stretched)



Elastic materials return to their original shape and size after stretching whilst plastic materials don't

- A change of shape is called a **deformation** and can either be:
  - Elastic
  - Plastic

Elastic Deformation

- Elastic deformation occurs:

When objects return to their original shape when the stretching force is removed

- Examples of materials that undergo elastic distortion are:
  - Rubber bands
  - Fabrics

- Steel springs

Your notes

Plastic Deformation

- Plastic deformation occurs:

When objects remain stretched and do not return completely to their original shape even when the stretching force is removed

- Examples of materials that undergo inelastic distortion are:
  - Plastic
  - Clay
  - Glass

2.5.2 Hooke's Law

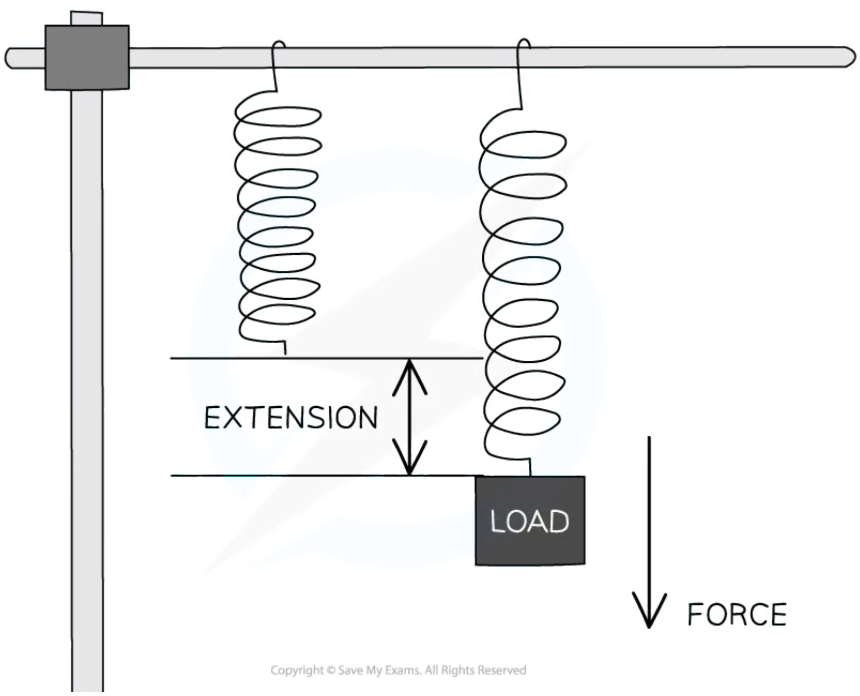
Your notes

Hooke's Law

- The relationship between the extension of an elastic object and the force applied is defined by **Hooke's Law**
- Hooke's Law states that:

The extension of an elastic object is directly proportional to the force applied, up to the limit of proportionality

- Directly proportional means that as the force is increased, the extension increases
  - If the force is doubled, then the extension will double
  - If the force is halved, then the extension will also halve
- The **limit of proportionality** is the point beyond which the relationship between force and extension is no longer directly proportional
  - This limit varies according to the material



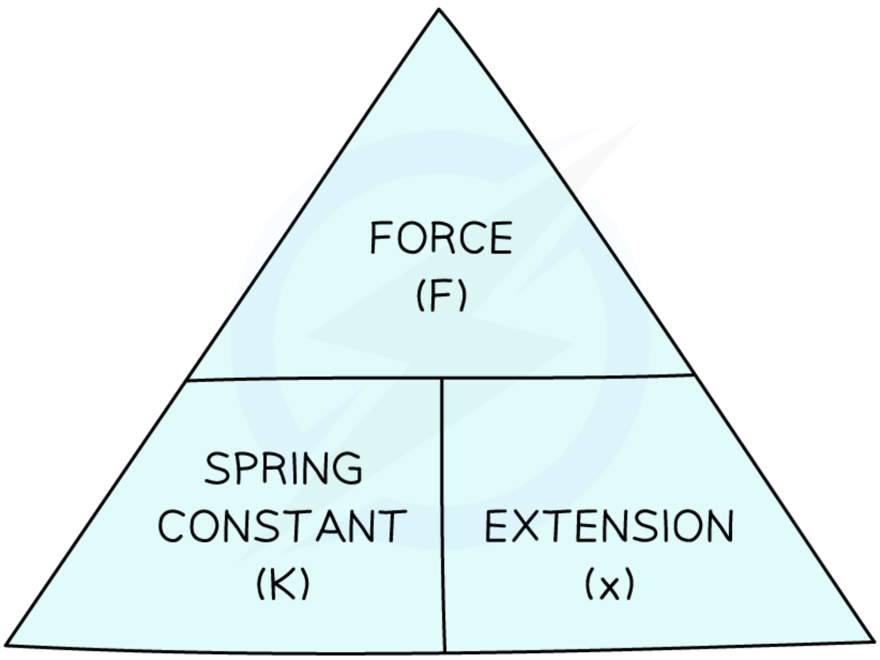
Hooke's Law states that a force applied to a spring will cause it to extend by an amount proportional to the force

- Hooke's Law is defined by the equation:

$$F = k \times x$$

- Where:
  - F = force in newtons (N)
  - k = spring constant in newtons per metres (N/m)
  - x = extension in metres (m)

- The symbol x can represent either the extension **or** compression of an elastic object
- The Hooke's law equation can be rearranged using the following formula triangle:



- The spring constant represents how **stiff** the spring is
  - The higher the **spring constant**, the higher the **stiffness**

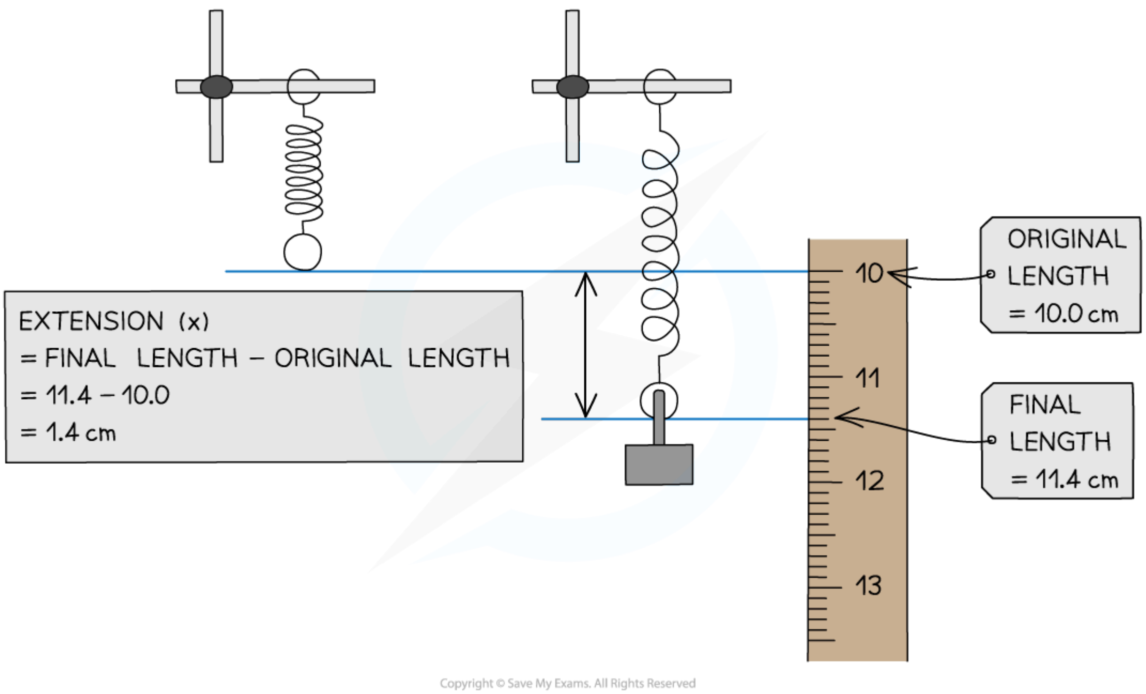
- The extension of an object can be calculated by:

Final length – Original length

- The extension of the spring can be measured by marking the position of bottom of the unstretched spring
- When the spring is stretched the final length must be measured from the bottom of the spring

Your notes

Your notes

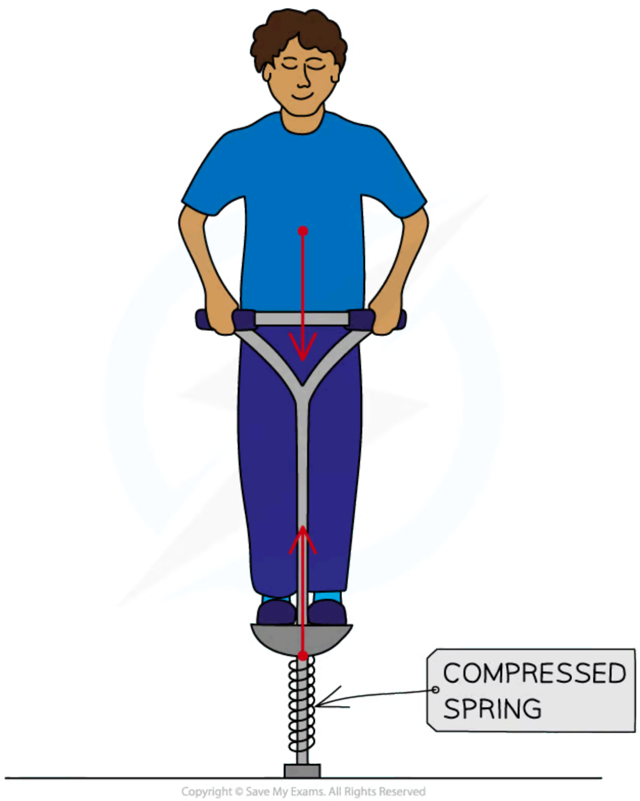


The extension measured from its final and original length

Your notes

Worked example

The figure below shows the forces acting on a child who is balancing on a pogo stick. The child and pogo stick are not moving.



The spring constant of the spring on the pogo stick is 4900 N/m. The weight of the child causes the spring to compress elastically from a length of 40 cm to a new length of 33 cm. Calculate the weight of the child.

Step 1: List the known quantities

- Spring constant, k = 4900 N/m
- Original length = 40 cm
- Final length = 33 cm

Step 2: Write the relevant equation

$$F = kx$$

Step 3: Calculate the extension, *x*

Your notes

$$x = \text{final length} - \text{original length} = 40 - 33 = 7 \text{ cm}$$

Step 4: Convert any units

- Since the spring constant is given in N/m, *x* must be in metres (m)

$$7 \text{ cm} = 0.07 \text{ m}$$

Step 5: Substitute the values into the Hooke's Law equation

$$F = 4900 \times 0.07 = \mathbf{343\ N}$$

Exam Tip

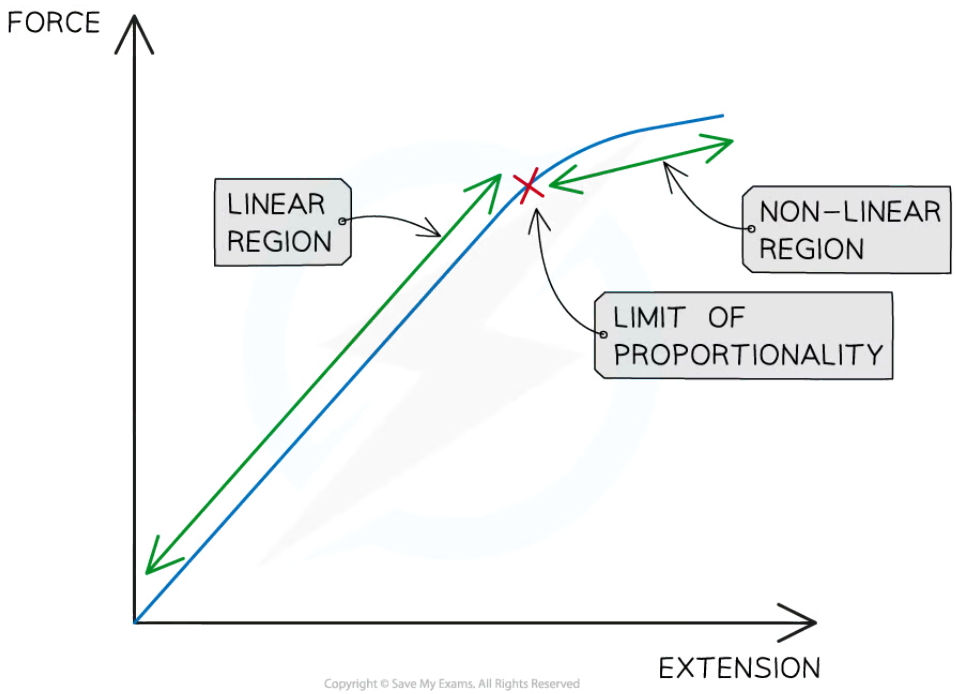
Look out for unit conversions! Unless the spring constant is given in N/cm, make sure the extension is converted into **metres** (÷ 100) before substituting values into the Hooke's Law equation.

2.5.3 Force-Extension Graphs

Your notes

Linear & Non-Linear Extension

- Hooke's law is the **linear** relationship between force and extension
  - This is represented by a **straight line** on a force-extension graph
- Materials that do **not** obey Hooke's law, i.e they do not return to their original shape once the force has been removed, have a **non-linear** relationship between force and extension
  - This is represented by a **curve** on a force-extension graph
- Any material beyond its limit of proportionality will have a non-linear relationship between force and extension



Linear and non-linear regions of a force-extension graph

Exam Tip

Remember these important mathematical terms:

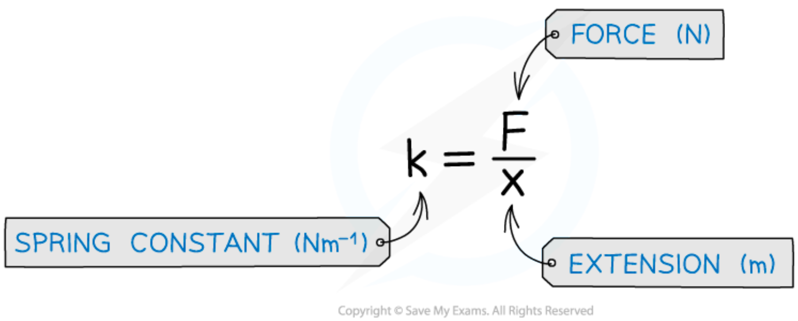
- **Proportional** = when a graph is a straight line going through the origin
- **Linear** = when a graph is a straight line (but does not necessarily go through the origin)
- **Non-linear** = when a graph is not a straight line

Your notes

Calculating the Spring Constant

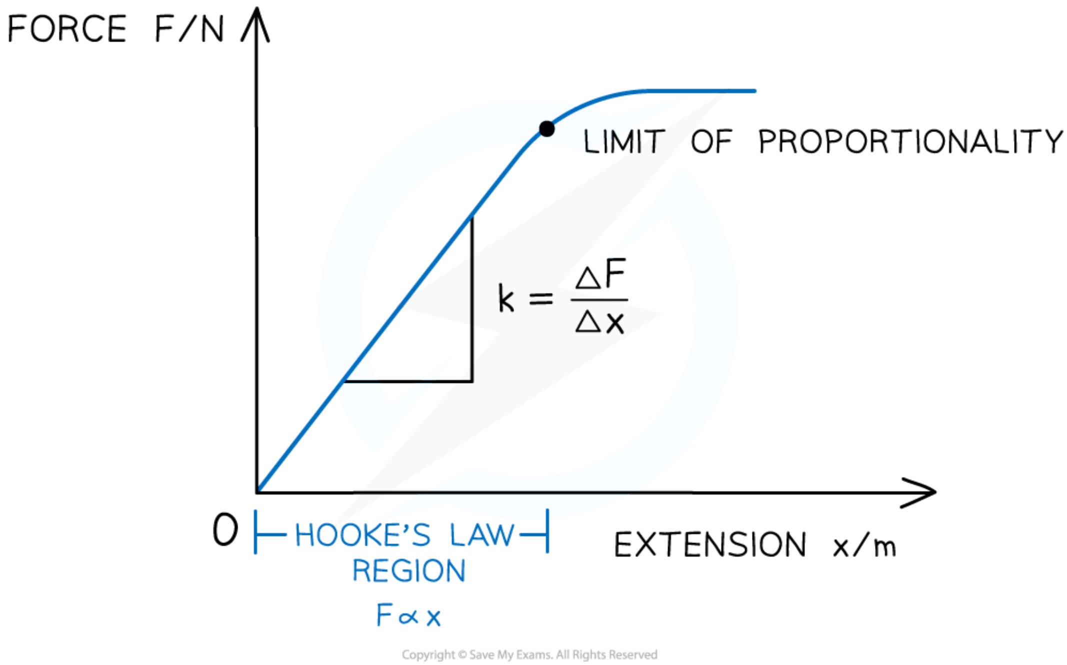
Your notes

- k is the **spring constant** of the spring and is a measure of the **stiffness** of a spring
  - A stiffer spring will have a larger value of k
- It is defined as the **force per unit extension** up to the limit of proportionality (after which the material will not obey Hooke's law)
- The SI unit for the spring constant is **N m^{-1}**
- Rearranging the Hooke's law equation shows the equation for the spring constant is



Spring constant equation

- The spring constant is the **force per unit extension** up to the limit of proportionality (after which the material will not obey Hooke's law)
- Therefore, the spring constant k is the gradient of the linear part of a Force v Extension graph



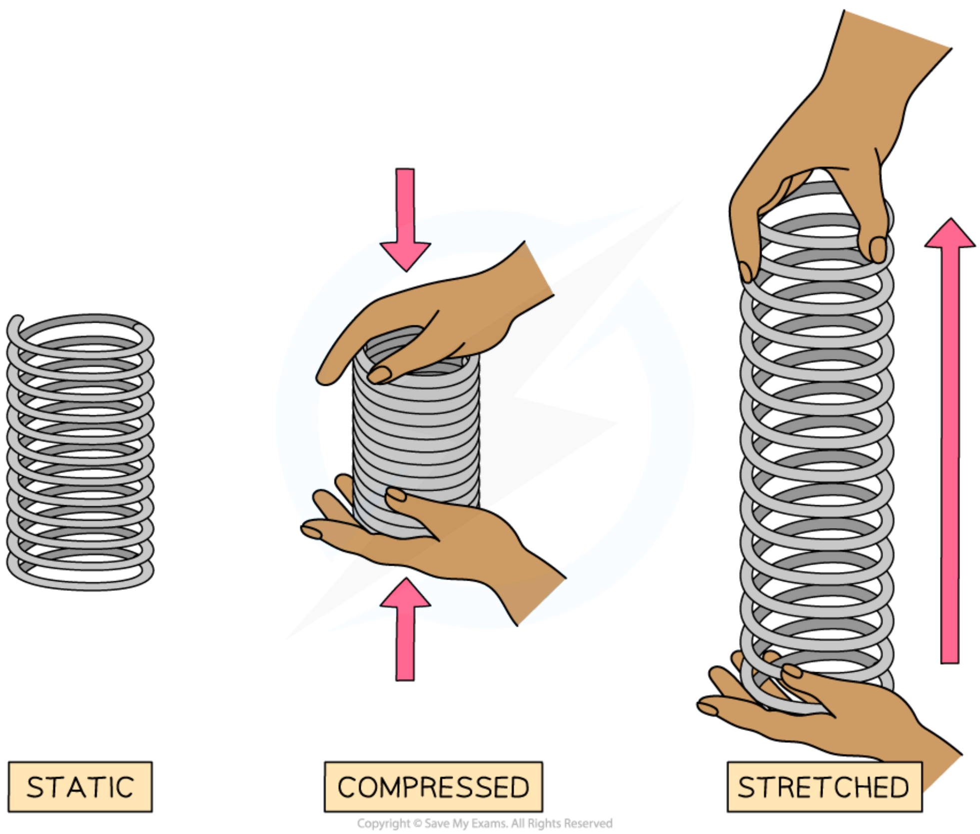
Spring constant is the gradient of a force v extension graph

2.5.4 Work Done on a Spring

Your notes

Work Done on a Spring

- When a spring is stretched or compressed by a force, **work** is done by the spring
- Work done is the transfer of energy
  - The energy is transferred to its **elastic potential energy** store



When a spring is stretched or compressed, there is work done and elastic potential energy is stored

- Elastic potential energy is defined as:

The energy stored in an elastic object when work is done on the object

- Provided the spring is not inelastically deformed (i.e has not exceeded its limit of proportionality), the work done on the spring and its elastic potential energy stored are **equal**

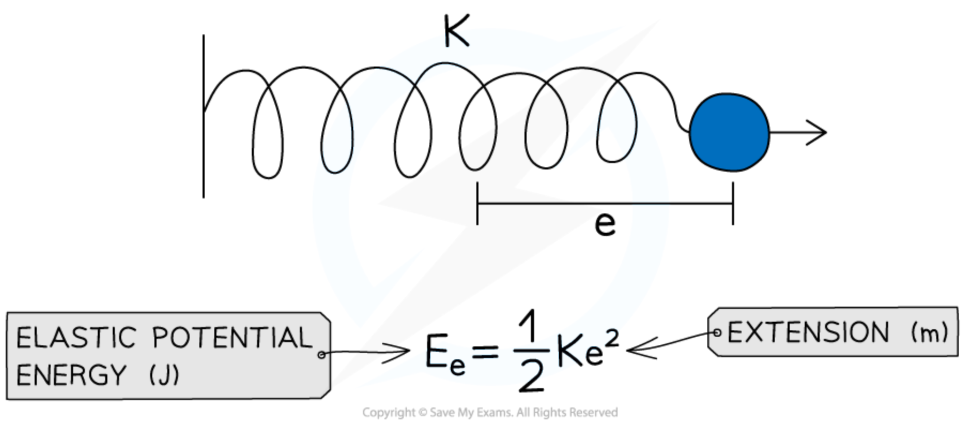
Calculating the Work Done on a Spring

Your notes

- The work done, or the elastic potential energy stored, while stretching or compressing a spring can be calculated using the equation:

$$E_e = ½ \times k \times e^2$$

- Where:
  - E_e = elastic potential energy in joules (J)
  - k = spring constant in newtons per metre (N/m)
  - e = extension in metres (m)



The elastic potential energy in a stretched spring depends on its spring constant and extension

- This equation is only for springs that have not been stretched beyond their **limit of proportionality**
  - The term e^2 means that if the extension is **doubled** then the work done is **quadrupled**
  - This is because $2^2 = 4$

Worked example

A mass is attached to the bottom of a hanging spring with a spring constant *k* and 0.2 J of work is done to stretch it by 4.5 cm. Calculate the spring constant, *k* for this spring.

Your notes

Step 1: List the known quantities

- Work done / elastic potential energy, E_e = 0.2 J
- Extension, e = 4.5 cm

Step 2: Write down the relevant equation

$$E_e = \frac{1}{2}ke^2$$

Step 3: Rearrange for the spring constant, *k*

- Multiply by 2 on both sides

$$2E_e = ke^2$$

- Divide by e^2 to make k the subject

$$\frac{2E_e}{e^2} = k$$

Step 4: Convert any units

- The extension should be in metres

$$4.5 \text{ cm} = 0.045 \text{ m}$$

Step 5: Substitute the values into the equation

$$k = \frac{2 \times 0.2}{(0.045)^2} = 198 \text{ N/m}$$

Exam Tip

Remember: when calculating the work done the extension, e, is **squared** (e^2)!

Sometimes you may see *x* instead of e. These are interchangeable so be prepared for both!

Your notes

2.5.5 PAG: Investigating Force & Extension

PAG 4: Investigating Force & Extension

Aim of the Experiment

- The aim of this experiment is to investigate the relationship between force and extension for a spring

Variables:

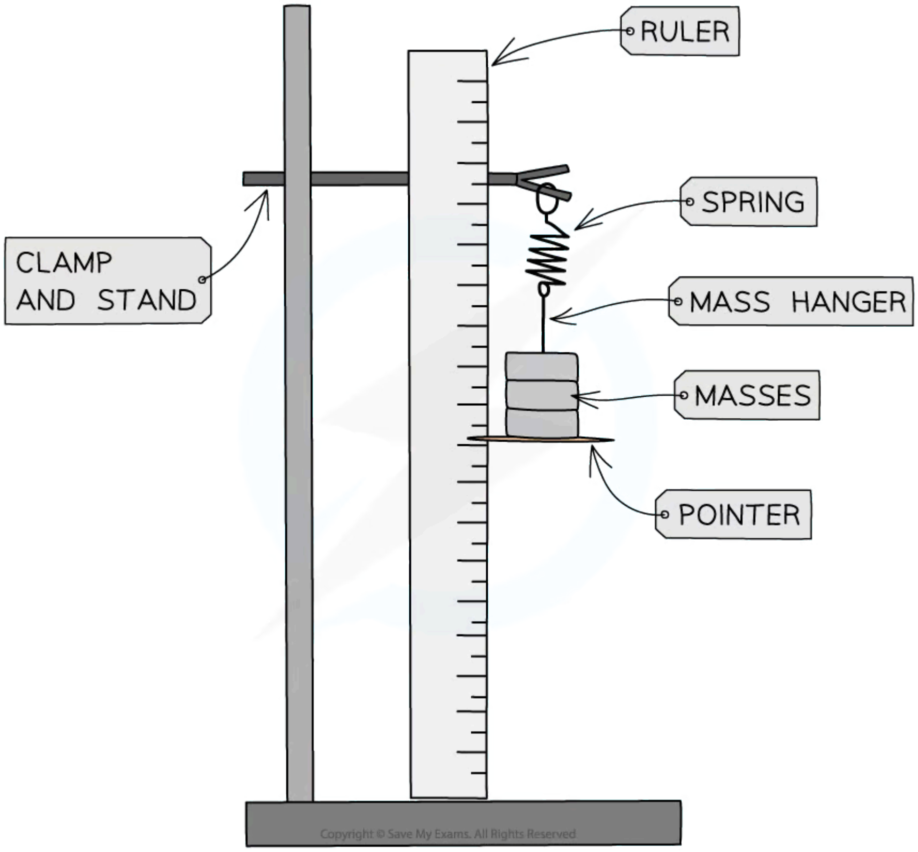
- Independent variable = Force, *F*
- Dependent variable = Extension, *x*
- Control variables:
  - Spring constant, *k*

Equipment List

Equipment	Purpose
Clamp and stand	To hold the spring and masses
Ruler	To measure the extension of the spring
Spring	To measure the extension of
5 × 100g masses	To apply a force to the spring
100g mass hanger	To hold the additional masses
Pointer (fiducial marker)	To accurately read the extension from the ruler

- Resolution of measuring equipment:
  - Ruler = 1 mm

Method



Investigating Hooke's law apparatus

1. **Set up** the apparatus as shown in the diagram, initially without any masses hanging from the spring
2. **Align the marker** to a value on the ruler, record the initial length of the spring
3. **Add** the **100 g mass hanger** onto the spring
4. **Record** the **mass** (in kg) and position (in cm) from the ruler now that the spring has extended
5. **Add another** 100 g to the mass hanger
6. **Record** the **new mass** and **position** from the ruler now that the spring has extended further
7. **Repeat** this process until all masses have been added
8. The masses are then removed and the entire process is repeated again until it has been carried out a total of three times, and an average length is calculated

- An example table of results might look like this:

Your notes

F = mg

AVERAGE LENGTH – ORIGINAL

MASS/kg	FORCE/N	LENGTH 1/m	LENGTH 2/m	LENGTH 3/m	AVERAGE LENGTH/m	EXTENSION/m
0						
0.1						
0.2						
0.3						
0.4						
0.5						
0.6						

Analysis of Results

- The force, *F* added to the spring is the **weight** of the mass
- The weight is calculated using the equation:

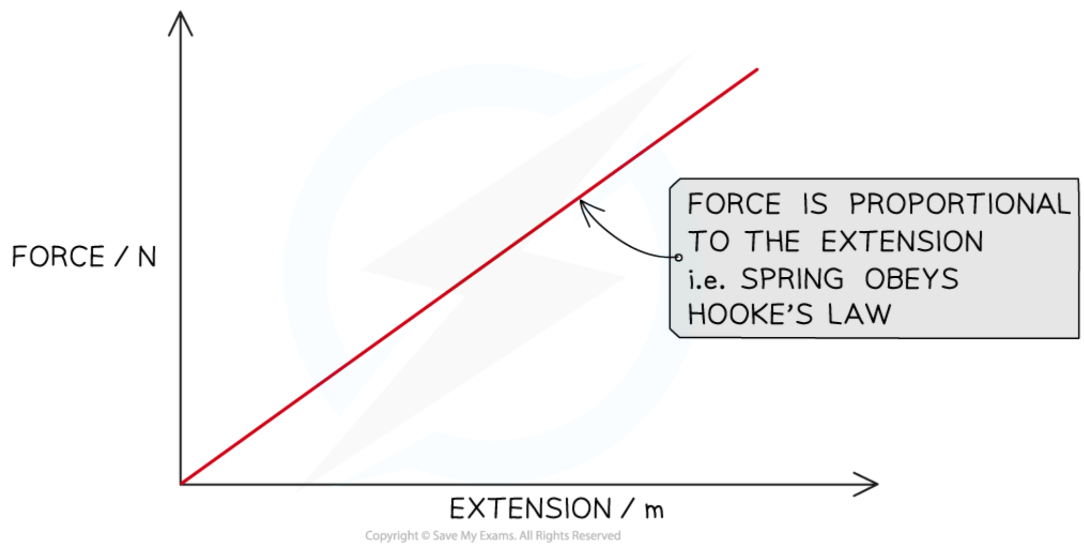
$$W = mg$$

- Where:
  - W = weight in newtons (N)
  - m = mass in kilograms (kg)
  - g = gravitational field strength on Earth in newtons per kg (N/kg)
- Therefore, **multiply** each mass by gravitational field strength, g, to calculate the force, F
  - The force can be calculated by multiplying the mass (in kg) by g = 10 N/kg
- The **extension** of the spring is calculated using the equation:

Final length – Original length

- The **final length** is the length of the spring recorded from the ruler when the masses were added
- The **original length** is the length of the spring when there were **no** masses

1. **Plot** a graph of the **force** against **extension**
2. Draw a line of best fit
3. If the graph has a **linear** region (is a straight line), then the force is **proportional** to the extension and the spring obeys Hooke's law for these forces and extension



Example force-extension graph for a spring that obeys Hooke's law

Your notes

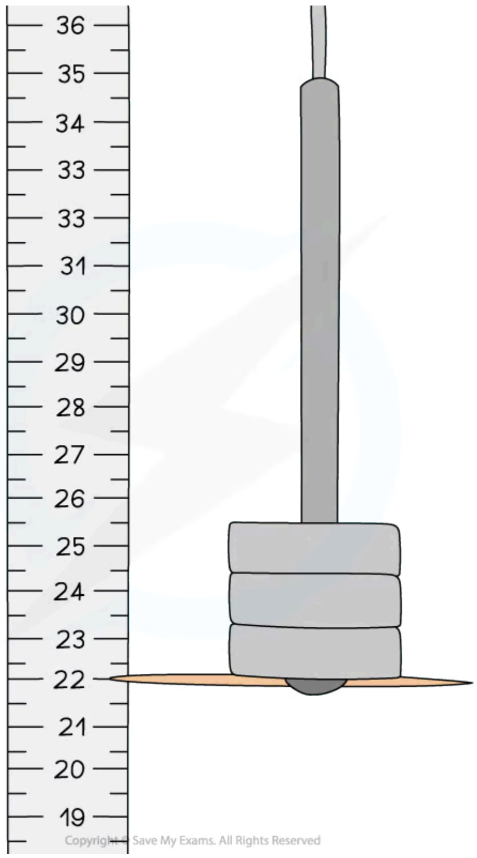
Evaluating the Experiment

Systematic Errors:

- Make sure the measurements on the ruler are taken at eye level to avoid **parallax error**

Random Errors:

- The accuracy of such an experiment is improved with the **use of a pointer** (a fiducial marker)



Fiducial marker to measure the extension more accurately

- **Wait** a few seconds for the spring to fully extend when a **mass is added**, before taking the reading for its new length
- Make sure to check whether the spring has not gone past its limit of proportionality otherwise, it has been stretched too far and will stop obeying Hooke's law

Safety Considerations

- Wear **goggles** during this experiment in case the spring snaps
- **Stand up** while carrying out the experiment making sure no feet are directly under the masses
- Place a **mat** or a soft material below the masses to prevent any damage in case they fall
- Use a **G clamp** to secure the clamp stand to the desk so that the clamp and masses do not fall over
  - As well as this, place each mass carefully on the hanger and do not pull the spring too hard that it breaks or pulls the apparatus over

Your notes

Exam Tip

Remember - the extension measures how much the object has stretched by and can be found by **subtracting the original length from each of the subsequent lengths**A common mistake is to calculate the increase in length by each time instead of the total extension – if each of your extensions is roughly the same then you might have made this mistake!

Your notes