Cambridge (CIE) IGCSE Maths: Extended

Your notes

Conditional Probability

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Conditional Probability

What is a conditional probability?

- A **conditional probability** is the probability of something happening (*A*) **given that** something else has **already happened** (*B*)



Examiner Tips and Tricks

Conditional probability can be written using formal notation.

The notation $\mathrm{P}(A \mid B)$ describes the probability of event A occurring, **given** that event B has already occurred.

Note that you are **not** expected to know this notation for the exam, but you are expected to be able to calculate conditional probabilities using Venn diagrams, tree diagrams or tables.

How do I calculate conditional probabilities?

- Conditional probabilities must be out of a **smaller restricted set** of outcomes (not out of all possible events)
  - For example, if a computer randomly selects a digit from 1, 2, 3, 4, 5, 6, 7, 8, 9
    - P(it selects a multiple of three) = $\frac{3}{9}$
    - This is **not** a conditional probability
    - There are 3 possibilities (3, 6, 9) out of **all** 9 possibilities
  - However, if you program the computer to only select from even numbers, then
    - P(it selects a multiple of three **given that** it selects an even number) = $\frac{1}{4}$
    - 1 possibility (6) out of **only** 4 possibilities (2, 4, 6, 8)
    - This **is** a conditional probability



Examiner Tips and Tricks

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- Look out for conditional probability questions within larger questions on two-way tables, Venn diagrams or tree diagrams
  - They often use the phrase **given that**

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Worked Example

A Venn diagram is shown below.

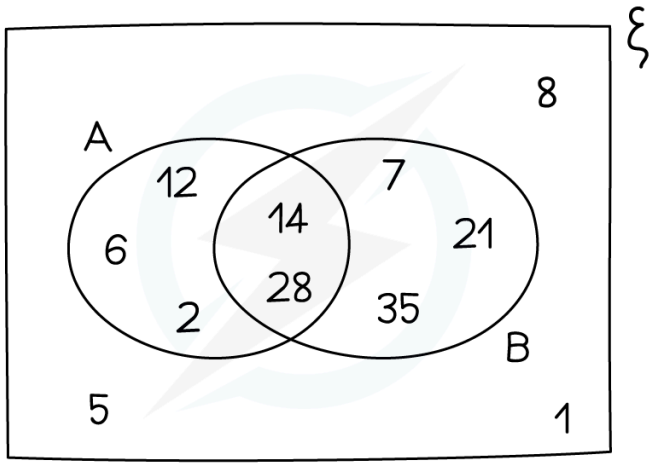
$A = \{2, 6, 12, 14, 28\}$

$B = \{7, 14, 21, 28, 35\}$

$\xi = \{1, 2, 5, 6, 7, 12, 14, 21, 28, 35\}$

A number is picked at random.

Find the probability that the number is in set A given that it is in set B.



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Answer:

Given that B has already happened, your probability will be out of number of numbers in *B*

Out of the 5 numbers in *B*, 2 of them are in *A*

Write as a fraction

$$\frac{2}{5}$$

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Combined conditional probabilities

What is a combined conditional probability?

- This is when you have two (or more) **successive events**, one after the other, and the **second** event **depends on** (is conditional on) the **first**

How do I calculate combined conditional probabilities?

- You need to **adjust** the number of outcomes as you go along
  - For example, selecting two cards from a pack of 52 playing cards **without replacing** the first card:
    - P(red 1st card) is 26 reds out of 52 cards
    - If the 1st card is not replaced, there are only 25 reds left out the remaining 51 cards
    - P(red 2nd card) is 25 reds out of 51 cards
    - P(red then red) = $\frac{26}{52} \times \frac{25}{51}$

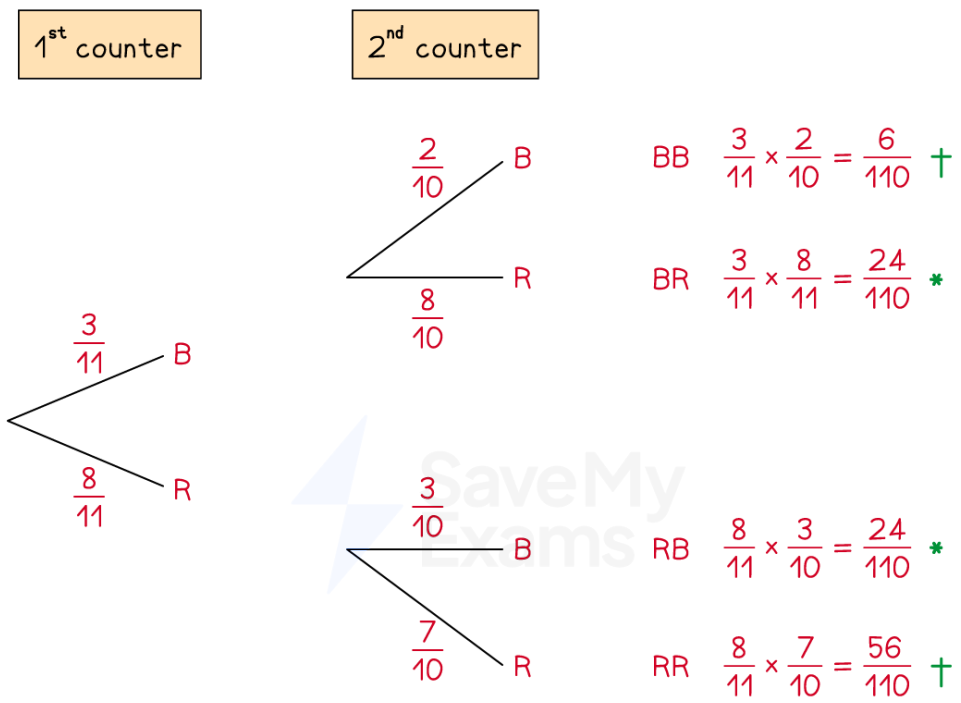
Examiner Tips and Tricks

If a question says "two cards are drawn" then you may **assume** that they draw 1 card followed by another card **without replacement** (the maths is the same).

Can I draw a tree diagram for combined conditional probabilities?

- Yes, a **tree diagram** is a useful way to show combined conditional probabilities
  - For example, two counters are drawn at random from a bag of 3 blue and 8 red counters without replacement
    - The probabilities are shown below

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* One of each colour: $\frac{24}{110} + \frac{24}{110} = \frac{48}{110}$

† Both same colour: $\frac{6}{110} + \frac{56}{110} = \frac{62}{110}$

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What if there are multiple possibilities within one question?

- You may need a **listing strategy** (e.g. *AAB*, *ABA*, *BAA*)
- You will need the **or** rule for multiple possibilities
  - P(*AB* **or** *BA* **or** *AA* **or**...) = P(*AB*) + P(*BA*) + P(*AA*) +...
    - **Add** the cases together
- Remember that *AB* and *BA* are **not the same**
  - *AB* means *A* happened first, then *B*
  - *BA* means *B* happened first, then *A*

Examiner Tips and Tricks

Try not to simplify your probabilities too early as it is easier to add probabilities together when they all have the same denominator!

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Worked Example

A bag contains 10 yellow beads, 6 blue beads and 4 green beads.

A bead is taken at random from the bag and not replaced.

A second bead is then taken at random from the bag.

(a) Find the probability that both beads are different colours.

Answer:

Let Y, B and G represent choosing a yellow, blue and green bead

Method 1

The probability of the beads being different colours is equal to 1 minus the probability that the beads are the same colour

Find the probability of both beads being the same colour

$$P(\text{same colours}) = P(YY) + P(BB) + P(GG)$$

Calculate each conditional probability separately, remembering the number of beads changes after one is drawn and not replaced

For example, $P(YY) = \frac{10}{20} \times \frac{9}{19}$

$$\frac{10}{20} \times \frac{9}{19} + \frac{6}{20} \times \frac{5}{19} + \frac{4}{20} \times \frac{3}{19}$$

Multiply the pairs of fractions together and add their results

$$\frac{232}{380}$$

Subtract this from 1

$$1 - \frac{132}{380} = \frac{248}{380}$$

Simplify the answer

$$\frac{62}{95}$$

Method 2

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List all the possibilities of different colours

Remember that YB (yellow first, then blue) is different to BY (blue first, then yellow)

YB, BY, YG, GY, BG, GB

Use the "or" rule to add the cases together

P(different colours) = P(YB) + P(BY) + P(YG) + P(GY) + P(BG) + P(GB)

Calculate each conditional probability separately, remembering the number of beads changes after one is drawn and not replaced

For example, P(YB) = $\frac{10}{20} \times \frac{6}{19}$

$$\frac{10}{20} \times \frac{6}{19} + \frac{6}{20} \times \frac{10}{19} + \frac{10}{20} \times \frac{4}{19} + \frac{4}{20} \times \frac{10}{19} + \frac{6}{20} \times \frac{4}{19} + \frac{4}{20} \times \frac{6}{19}$$

Multiply the pairs of fractions together and add their results

$$\frac{248}{380}$$

Simplify the answer

$$\frac{62}{95}$$

(b) The second bead is not replaced and a third bead is taken at random from the bag.

Find the probability that all three beads are the same colour.

Answer:

List the possibilities

YYY, BBB, GGG

Use the "or" rule to add between cases

P(all the same colour) = P(YYY) + P(BBB) + P(GGG)

Use conditional probabilities in each separate case, remembering the number of beads changes after each one is drawn and not replaced

$$\frac{10}{20} \times \frac{9}{19} \times \frac{8}{18} + \frac{6}{20} \times \frac{5}{19} \times \frac{4}{18} + \frac{4}{20} \times \frac{3}{19} \times \frac{2}{18}$$

Multiply the triplets of fractions together then add their results

$$\frac{720}{6840} + \frac{120}{6840} + \frac{24}{6840} = \frac{864}{6840}$$

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Simplify the answer

$$\frac{12}{95}$$

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Worked Example

A bag contains only red and blue marbles. There are 3 more red marbles than blue marbles.

Kai takes 2 marbles from the bag at random.

The probability that Kai takes 2 blue marbles is $\frac{1}{7}$.

Work out the number of blue marbles there were in the bag initially.

Show clear algebraic working.

Answer:

Write expressions for the number of each colour

Let x be the number of blue marbles

Then there are $x + 3$ red marbles

And there are $x + x + 3 = 2x + 3$ marbles in total

Find the probability that the first marble is blue

$$\frac{x}{2x+3}$$

Given that the first marble is blue, find the probability that the second marble is blue

- After one blue marble is taken, there are $x - 1$ blue marbles left out of $2x + 2$ marbles in total

$$\frac{x-1}{2x+2}$$

Find the probability that both marbles are blue by multiplying the probabilities together

- Set this equal to $\frac{1}{7}$

$$\frac{x}{2x+3} \times \frac{x-1}{2x+2} = \frac{1}{7}$$

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Multiply through by $7(2x+3)(2x+2)$ to get rid of the fractions

$$7x(x-1)=(2x+3)(2x+2)$$

Expand and simplify both sides

$$7x^2-7x=4x^2+4x+6x+6$$
$$7x^2-7x=4x^2+10x+6$$

Rearrange to make one side equal to zero

$$3x^2-17x-6=0$$

Factorise the quadratic

- Find two numbers that multiply to give 3×-6=-18 and add to give -17
- 1 and -18

$$3x^2+x-18x-6$$
$$x(3x+1)-6(3x+1)$$
$$(3x+1)(x-6)$$

Solve the quadratic equation

$$(3x+1)(x-6)=0$$
$$x=-\frac{1}{3},\ x=6$$

x needs to be a positive whole number

$x=6$ blue marbles initially



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