Cambridge (CIE) A Level Physics

Your notes

Mass Defect & Nuclear Binding Energy

Contents

Energy & mass equivalence

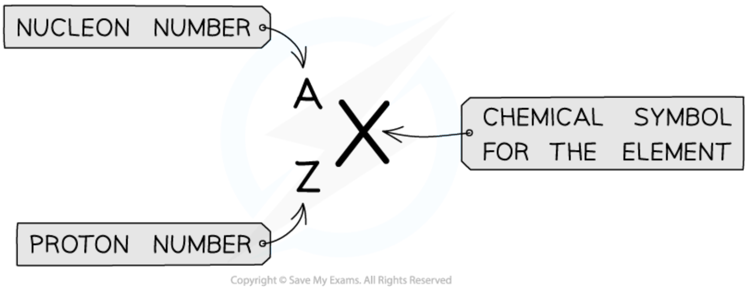
- Einstein showed in his **theory of relativity** that matter can be considered a form of energy and hence, he proposed:
  - mass can be converted into energy
  - energy can be converted into mass
- This is known as **mass-energy equivalence**, and can be summarised by the equation:

$$E = mc^2$$

- Where:
  - E = energy (J)
  - m = mass (kg)
  - c = the speed of light (m s^{-1})
- Some examples of mass-energy equivalence are:
  - the **fusion** of hydrogen into helium in the centre of the sun
  - the **fission** of uranium in nuclear power plants
  - nuclear **weapons**
  - high-energy **particle collisions** in particle accelerators

Representing simple nuclear reactions

- A nucleus can be described using $^{A}_{Z}X$ **notation**



$^{A}_{Z}X$ notation is used to describe the constituents of a nucleus

- The top number A represents the **nucleon** number or the **mass** number
  - **Nucleon number (A)** = total number of **protons and neutrons** in the nucleus
- The lower number Z represents the **proton** or **atomic** number
  - **Proton number (Z)** = total number of **protons** in the nucleus

Worked Example

When a neutron is captured by a uranium-235 nucleus, the outcome may be represented by the nuclear equation:

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{95}_{42}\mathrm{Mo} + ^{139}_{57}\mathrm{La} + x\,^{1}_{0}\mathrm{n} + 7\,^{0}_{-1}\mathrm{e}$$

Determine the value of x.

Answer:

Step 1: Balance the nucleon numbers (the top number)

$$235 + 1 = 95 + 139 + x(1) + 7(0)$$

Step 2: Rearrange to find the value of x

$$x = 235 + 1 - 95 - 139 = 2$$

Mass defect & binding energy

Mass defect

- Experiments into nuclear structure have found that the total mass of a nucleus is **less** than the sum of the masses of its constituent nucleons
- This difference in mass is known as the **mass defect**
- Mass defect is defined as:

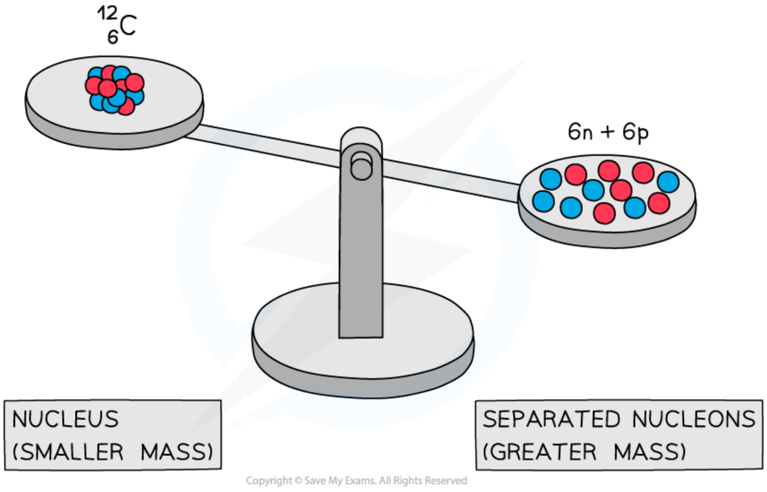
The difference between the mass of a nucleus and the sum of the individual masses of its protons and neutrons

- The mass defect Δm of a nucleus can be calculated using:

$$\Delta m = Zm_p + (A - Z)m_n - m_{total}$$

- Where:
  - Z = proton number
  - A = nucleon number
  - m_p = mass of a proton (kg)
  - m_n = mass of a neutron (kg)
  - m_{total} = measured mass of the nucleus (kg)

Mass defect of carbon-12



A system of separated nucleons has a greater mass than a system of bound nucleons

- Due to the equivalence of mass and energy, this decrease in mass implies that energy is released in the process
- Since nuclei are made up of neutrons and protons, there are forces of repulsion between the positive protons
    - Therefore, it takes energy, i.e. the binding energy, to hold nucleons together as a nucleus

Binding energy

- Binding energy is defined as:

  The energy required to break a nucleus into its constituent protons and neutrons

- Energy and mass are proportional, so the total energy of a nucleus is **less** than the sum of the energies of its constituent nucleons
- The formation of a nucleus from a system of isolated protons and neutrons is therefore an exothermic reaction
    - This means that it **releases energy**
- This energy can be calculated using the equation:

$$E = \Delta mc^2$$

- In a typical nucleus, binding energies are usually measured in **MeV**
    - This is considerably larger than the few eV associated with the binding energy of electrons in the atom
- Nuclear reactions involve changes in the nuclear binding energy whereas chemical reactions involve changes in the electron binding energy
    - This is why nuclear reactions produce **much more energy** than chemical reactions

Worked Example

Calculate the binding energy per nucleon, in MeV, for the radioactive isotope potassium-40 ($_{19}$K).

- Nuclear mass of potassium-40 = 39.953 548 u
- Mass of one neutron = 1.008 665 u
- Mass of one proton = 1.007 276 u

Answer:

Step 1: Identify the number of protons and neutrons in potassium-40

- Proton number, Z = 19
- Neutron number, N = 40 – 19 = 21

Step 2: Calculate the mass defect, Δ*m*

- Proton mass, m_p = 1.007 276 u

- Neutron mass, $m_n = 1.008\,665$ u
- Mass of K-40, $m_{total} = 39.953\,548$ u

$$\Delta m = Zm_p + Nm_n - m_{total}$$

$$\Delta m = (19 \times 1.007276) + (21 \times 1.008665) - 39.953\,548$$

$$\Delta m = 0.36666 \text{ u}$$

Step 3: Convert from u to kg

- $1 \text{ u} = 1.661 \times 10^{-27}$ kg

$$\Delta m = 0.36666 \times (1.661 \times 10^{-27}) = 6.090 \times 10^{-28} \text{ kg}$$

Step 4: Write down the equation for mass-energy equivalence

$$E = \Delta mc^2$$

- Where c = speed of light

Step 5: Calculate the binding energy, *E*

$$E = 6.090 \times 10^{-28} \times (3.0 \times 10^8)^2 = 5.5 \times 10^{-11} \text{ J}$$

Step 6: Determine the binding energy per nucleon and convert J to MeV

- Take the binding energy and divide it by the number of nucleons
- $1 \text{ MeV} = 1.6 \times 10^{-13}$ J

$$\text{binding energy per nucleon} = \frac{5.5 \times 10^{-11}}{40} = 1.375 \times 10^{-12} \text{ J}$$

$$\text{binding energy per nucleon} = \frac{1.375 \times 10^{-12}}{1.6 \times 10^{-13}} = 8.594 \text{ MeV}$$

Examiner Tips and Tricks

The terms binding energy and mass defect can cause students confusion, so be careful when using them in your explanations.

Avoid describing the binding energy as the energy stored in the nucleus – this is not correct – it is energy that must be put **into** the nucleus to separate **all** the nucleons.

The same goes for the term mass defect, make sure to only use this when all the nucleons are separated and not to describe the decrease in mass which occurs during radioactive decay.

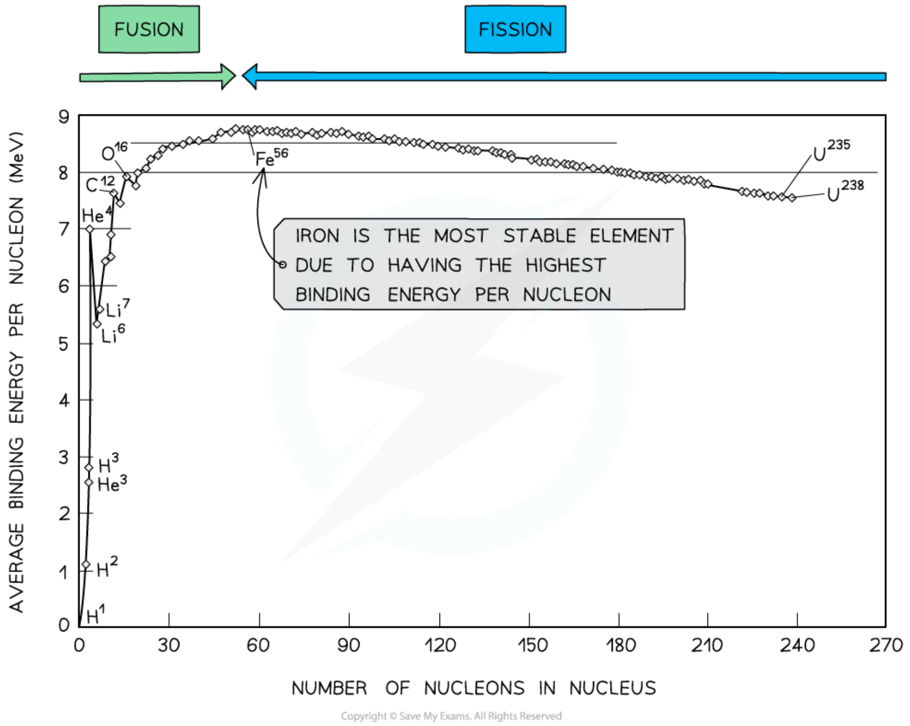
Binding energy per nucleon

- In order to compare nuclear stability, it is more useful to look at the **binding energy per nucleon**
- The binding energy per nucleon is defined as:

The binding energy of a nucleus divided by the number of nucleons in the nucleus

- A higher binding energy per nucleon indicates a higher stability
  - In other words, more energy is required to separate the nucleons contained within a nucleus

Graph of binding energies for nuclei of different masses



By plotting a graph of binding energy per nucleon against nucleon number, the stability of elements can be inferred

Key features of the graph

- At low values of A (nucleon number):
  - Nuclei have lower binding energies per nucleon than at large values of A, but they tend to be stable when N = Z (proton number)
  - This means light nuclei have weaker electrostatic forces and will undergo **fusion**
  - The gradient is **much steeper** compared to the gradient at large values of A
  - This means that fusion reactions release a greater binding energy than fission reactions
- At high values of A (nucleon number):
  - Nuclei have generally higher binding energies per nucleon, but this gradually decreases with A

    - This means the heaviest elements are the most unstable and will undergo **fission**
    - The gradient is **less steep** compared to the gradient at low values of A
    - This means that fission reactions release less binding energy than fusion reactions
- Iron (A = 56) has the highest binding energy per nucleon, which makes it the **most stable** of all the elements
- Helium (^{4}He), carbon (^{12}C) and oxygen (^{16}O) do not fit the trend
    - Helium-4 is a particularly stable nucleus, hence it has a **high** binding energy per nucleon
    - Carbon-12 and oxygen-16 can be considered to be three and four helium nuclei, respectively, bound together

Worked Example

Determine the binding energy per nucleon of iron-56, $^{56}_{26}\text{Fe}$, in MeV.

- Mass of a neutron = 1.675×10^{-27} kg
- Mass of a proton = 1.673×10^{-27} kg
- Mass of an iron-56 nucleus = 9.288×10^{-26} kg

Answer:

Step 1: Calculate the mass defect

- Number of protons, Z = 26
- Number of neutrons, A – Z = 56 – 26 = 30

Mass defect: $\Delta m = Zm_p + (A - Z)m_n - m_{total}$

$$\Delta m = (26 \times 1.673 \times 10^{-27}) + (30 \times 1.675 \times 10^{-27}) - (9.288 \times 10^{-26})$$

$$\Delta m = 8.680 \times 10^{-28} \text{ kg}$$

Step 2: Calculate the binding energy of the nucleus

Binding energy: $E = \Delta mc^2$

$$E = (8.680 \times 10^{-28}) \times (3.00 \times 10^8)^2 = 7.812 \times 10^{-11} \text{ J}$$

Step 3: Calculate the binding energy per nucleon

Binding energy per nucleon = $\frac{E}{A}$

$$\frac{E}{A} = \frac{7.812 \times 10^{-11}}{56} = 1.395 \times 10^{-12} \text{ J}$$

Step 4: Convert to MeV

- J → eV: divide by 1.6×10^{-19}

- eV → MeV: divide by 10^6

$$\text{Binding energy per nucleon} = \frac{1.395 \times 10^{-12}}{1.6 \times 10^{-19}}$$

$$\text{Binding energy per nucleon} = 8\ 718\ 750\ \text{eV} = 8.7\ \text{MeV}\ (2\ \text{s.f.})$$

Your notes

Examiner Tips and Tricks

Checklist on what to include (and what not to include) in an exam question asking you to draw a graph of binding energy per nucleon against nucleon number:

- You will be expected to draw the best fit curve AND a cross to show the anomaly that is helium
- Do not begin your curve at A = 0, this is not a nucleus!
- Make sure to correctly label both axes AND units for binding energy per nucleon
- You will be expected to include numbers on the axes, mainly at the peak to show the position of iron (^{56}Fe)

Nuclear fusion & fission

Nuclear fusion

- Nuclear fusion is when:

Two nuclei combine to form a single nucleus

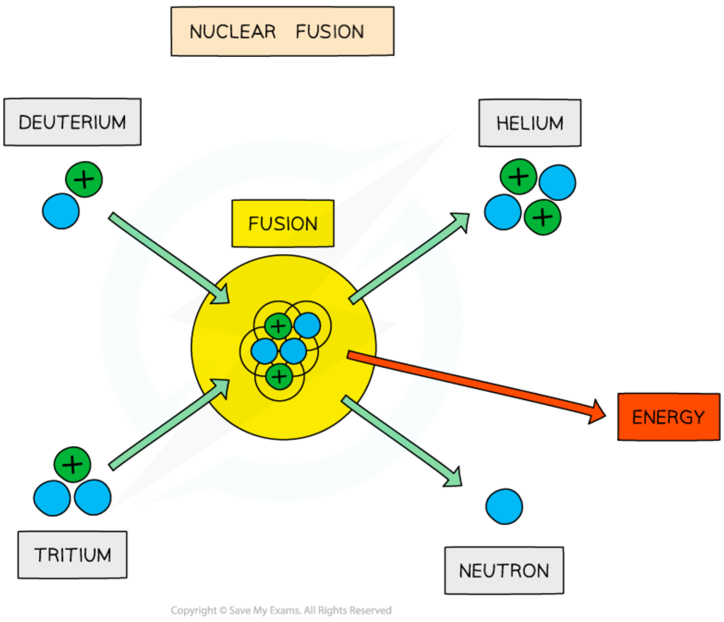
- Low mass nuclei, such as hydrogen and helium, can undergo fusion and release energy
- For example, when two hydrogen nuclei (protons) fuse, a deuterium nucleus is produced
  - A positron and an electron neutrino are also produced as one of the protons converts into a neutron through beta-plus decay

$$ {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + e^{+} + \nu_{e} $$

- In the centres of stars, four hydrogen nuclei $\left({}^{1}_{1}\mathrm{H}\right)$ fuse to produce a helium nucleus $\left({}^{4}_{2}\mathrm{He}\right)$, plus the release of energy
  - This provides fuel for the star to continue burning
- On Earth, research is focused on achieving the deuterium-tritium (D-T) reaction
- This involves fusing a deuterium nucleus and a tritium nucleus together to produce a helium nucleus and a neutron

$$ {}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} $$

Deuterium-tritium fusion



The fusion of deuterium and tritium nuclei to form a helium nucleus and a neutron, with the release of energy

- For two nuclei to fuse, both nuclei must have **high kinetic energy**
- This is because nuclei must be able to get close enough to fuse
- However, two forces acting within the nuclei make this difficult to achieve
    - **Electrostatic repulsion**
        - Protons inside the nuclei are positively charged, which means that they **electrostatically** repel one another
    - **Strong nuclear force**
        - The strong nuclear force, which binds nucleons together, acts at very short distances within nuclei
        - Therefore, nuclei must get very close together for the strong nuclear force to take effect
- It takes a great deal of energy to overcome the electrostatic force, hence, fusion can only be achieved in an **extremely hot** environment, such as the core of a star

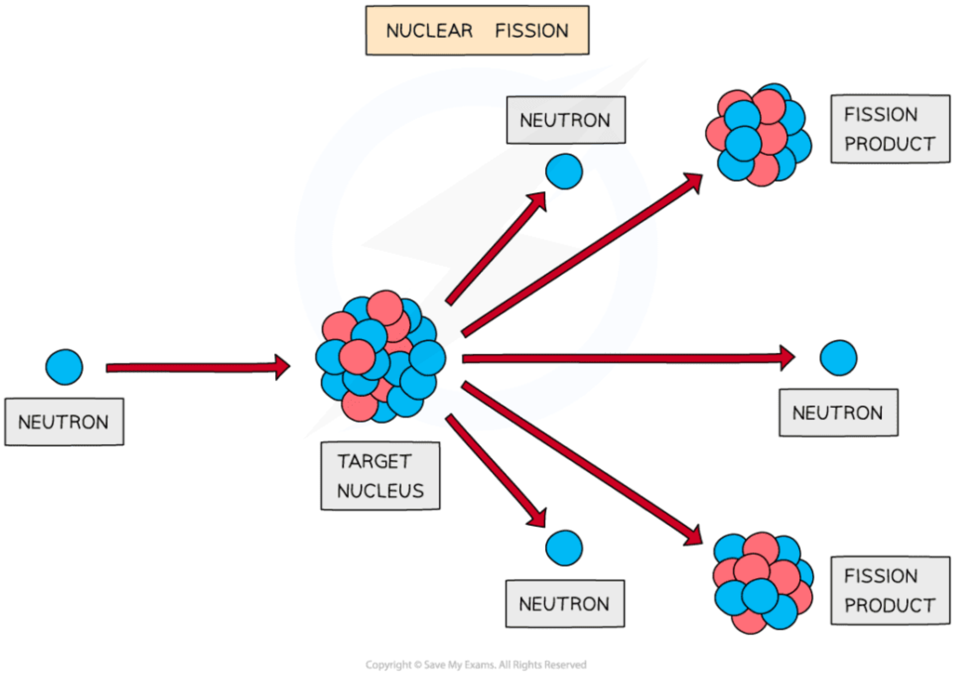
Nuclear fission

- Nuclear fission is when:

A single large nucleus divides to form smaller nuclei

- High mass nuclei (such as uranium) can undergo fission and release energy

Induced fission



The fission of a target nucleus, such as uranium, to produce smaller daughter nuclei, with the release of energy

- Fission must be **induced** by firing neutrons at a nucleus
    - When a neutron strikes a nucleus, it splits into two or more daughter nuclei
    - During fission, neutrons are ejected from the nucleus, which in turn can collide with other nuclei, triggering a cascade effect
    - This leads to a chain reaction, which lasts until all of the material has undergone fission, or the reaction is halted by a moderator
- Nuclear fission is the process which produces energy in nuclear power stations, where it is well controlled
- When nuclear fission is not controlled, the chain reaction can cascade to produce the effects of a nuclear bomb

Examiner Tips and Tricks

When an atom undergoes nuclear fission, take note that extra neutrons are ejected by the **nucleus** and not from the fission products

Significance of binding energy per nucleon

- At low values of A:

  - attractive nuclear forces between nucleons dominate over repulsive electrostatic forces between protons
  - in the right conditions, nuclei undergo **fusion**
- In fusion, the mass of the nucleus that is created is slightly **less** than the total mass of the original nuclei
  - The mass defect is equal to the binding energy that is released since the nucleus that is formed is more stable
- At high values of A:
  - repulsive electrostatic forces between forces begin to dominate, and these forces tend to break apart the nucleus rather than hold it together
  - in the right conditions, nuclei undergo **fission**
- In fission, an unstable nucleus is converted into more stable nuclei with a smaller total mass
  - This difference in mass, the mass defect, is equal to the binding energy that is released

Calculating energy released in nuclear reactions

- The binding energy is equal to the amount of energy released in forming the nucleus, and can be calculated using:

$$E = (\Delta m)c^2$$

- Where:
  - E = Binding energy released (J)
  - Δm = mass defect (kg)
  - c = speed of light (m s^{-1})
- The daughter nuclei produced as a result of both fission and fusion have a higher binding energy per nucleon than the parent nuclei
- Therefore, energy is released as a result of the mass difference between the parent nuclei and the daughter nuclei

Worked Example

When uranium-235 nuclei undergo fission by absorbing slow-moving neutrons, two reactions are possible:

$$\text{Reaction 1: } {}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{139}_{54}\text{Xe} + {}^{95}_{38}\text{Sr} + 2\,{}^{1}_{0}\text{n} + \text{energy}$$

$$\text{Reaction 2: } {}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow 2\,{}^{116}_{46}\text{Pd} + xc + \text{energy}$$

(a) For reaction 2, identify the particle c, and state the number, x, of such particles produced in the reaction.

(b) The binding energy per nucleon, E, for a number of nuclides is given by the table below. Use the table to show that the energy produced in reaction 1 is about 210 MeV.

(c) The energy produced in reaction 2 is 163 MeV. Suggest, with supporting reason, which one of the two reactions is more likely to happen.

nuclide	E / MeV
${}^{95}_{38}\text{Sr}$	8.74
${}^{139}_{54}\text{Xe}$	8.39

$^{235}_{92}U$	7.60

Answer:

Part (a)

Step 1: Balance the number of protons on each side (bottom number)

$$92 = (2 \times 46) + xn_p \text{ (where } n_p \text{ is the number of protons in c)}$$

$$xn_p = 92 - 92 = 0$$

Therefore, c must be a neutron

Step 2: Balance the number of nucleons on each side

$$235 + 1 = (2 \times 116) + x$$

$$x = 235 + 1 - 232 = 4$$

- Therefore, 4 neutrons are generated in the reaction

Part (b)

Step 1: Find the binding energy of each nucleus

Total binding energy of each nucleus = Binding energy per nucleon × Mass number

$$\text{Binding energy of } ^{95}\text{Sr} = 8.74 \times 95 = 830.3 \text{ MeV}$$

$$\text{Binding energy of } ^{139}\text{Xe} = 8.39 \times 139 = 1166.21 \text{ MeV}$$

$$\text{Binding energy of } ^{235}\text{U} = 7.60 \times 235 = 1786 \text{ MeV}$$

Step 2: Calculate the difference in energy between the products and reactants

$$\text{Energy released in reaction 1} = E_{Sr} + E_{Xe} - E_U$$

$$\text{Energy released in reaction 1} = 830.3 + 1166.21 - 1786$$

$$\text{Energy released in reaction 1} = 210.5 \text{ MeV}$$

Part (c)

- Since reaction 1 releases more energy than reaction 2, its end products will have a higher binding energy per nucleon
  - Hence they will be more stable
- This is because the more energy is released, the further it moves up the graph of binding energy per nucleon against nucleon number (A)
  - Since at high values of A, the binding energy per nucleon gradually decreases with A
- Nuclear reactions will tend to favour the more stable route, therefore, reaction 1 is more likely to happen