CIE IGCSE Maths: Core

Your notes

Linear Equations

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Linear Equations

Your notes

Solving Linear Equations

What are linear equations?

- A linear equation is an equation that will produce a straight line when plotted on a graph
- The greatest power of x in a linear equation is 1
  - This means there are no terms of x^2 or a higher order
- A linear equation is normally in form $ax + b = c$
  - where a, b, and c are constants and x is a variable

How do I solve a linear equation?

- To solve a linear equation you need to isolate the variable, usually x, by carrying out **inverse operations** to **both sides** of the equation
  - Inverse operations are just the opposite operations to what has already happened to the variable
- The order in which the inverse operations are carried out is important
  - Most of the time, this will be BIDMAS in reverse
  - However it depends on the order in which the operations were applied to the variable to form the equation

How do I solve a linear equation of the form *ax* + *b* = *c*?

- The operations that **have been applied** to x here are:
  - STEP 1
    Multiply by a
  - STEP 2
    Add b
- To solve this, you must carry out the inverse operations in **reverse order**
  - STEP 1
    Subtract b
  - STEP 2
    Divide by a
- For example, to solve the equation $2x + 1 = 9$
  - STEP 1
    Subtract 1

$$2x + 1 = 9$$

$$(-1) \qquad\qquad (-1)$$

$$2x = 8$$

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- STEP 2
  Divide by 2

$$2x = 8$$

$$(\div 2) \qquad (\div 2)$$

$$x = 4$$

- Be extra careful if any of the terms have negatives
- For example, to solve the equation $2 - 3x = 10$
  - STEP 1
    Subtract 2

$$2 - 3x = 10$$

$$(-2) \qquad (-2)$$

$$-3x = 8$$

    - Be careful not to drop the negative sign
  - STEP 2
    Divide by -3

$$-3x = 8$$

$$(\div -3) \qquad (\div -3)$$

$$x = -\frac{8}{3}$$

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Examiner Tip

- If you have time in the exam, you should substitute your answer back into the equation to check you got it right

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Worked example

Solve the equation

$$5x - 8 = 22$$

Add 8 to both sides of the equation

$$5x = 22 + 8$$

Work out 22 + 8

$$5x = 30$$

Divide both sides by 5

$$x = \frac{30}{5}$$

Work out 30 ÷ 5
Keep the x on the left-hand side

$$x = 6$$

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Equations with Brackets & Fractions


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How do I solve a linear equation that contains brackets?

- If a linear equation contains brackets on one, or both, sides start by expanding the brackets
- For example, to solve the equation $2(x - 1) = 10 - (3 + x)$
  - STEP 1
    Expand the brackets on both sides
    $$2x - 2 = 10 - 3 - x$$
  - STEP 2
    Collect the x terms to one side by subtracting the term with the smaller coefficient of x
    $$2x - 2 = 7 - x$$
    $$-(-x) \qquad\qquad -(-x)$$
    $$3x - 2 = 7$$
    - Be extra careful if any of the terms have negatives
  - STEP 3
    Solve the linear equation using the method above
    $$3x - 2 = 7$$
    $$3x = 9$$
    $$x = 3$$

How do I solve a linear equation that contains fractions?

- If a linear equation contains a fraction on one or both sides, remove the fractions by multiplying both sides by everything on the denominator
  - Remember to put brackets around the expression first
- For example, to solve the equation $\frac{2}{x + 3} = \frac{3}{4 - 2x}$ you will need to multiply both sides of the equation by both $(x + 3)$ and $(4 - 2x)$
  - STEP 1
    Multiply both sides by $(x + 3)$
    $$\frac{2}{x + 3}(x + 3) = \frac{3}{4 - 2x}(x + 3)$$
    $$2 = \frac{3(x+3)}{4 - 2x}$$
  - STEP 2
    Multiply both sides by $(4 - 2x)$

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$$2(4 - 2x) = \frac{3(x + 3)}{4 - 2x}(4 - 2x)$$
$$2(4 - 2x) = 3(x + 3)$$

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- STEP 3
  Expand the brackets on both sides
$$8 - 4x = 3x + 9$$
- STEP 4
  Collect the x terms to one side by subtracting the term with the smaller coefficient of x
$$8 - 4x = 3x + 9$$
$$-(-4x) \qquad\qquad -(-4x)$$
$$8 = 7x + 9$$
- STEP 5
  Solve the equation
  - You can swap the sides if it makes solving the equation easier
$$7x + 9 = 8$$
$$7x = -1$$
$$x = -\frac{1}{7}$$

Examiner Tip

- If you have time in the exam, you should substitute your answer back into the equation to check you got it right

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Worked example

a) Solve the equation.

$$\frac{3x - 2}{4 - x} = -1$$

Get rid of the fraction by multiplying both sides by the denominator $(4 - x)$

$$3x - 2 = -(4 - x)$$

Expand the brackets

$$3x - 2 = -4 + x$$

Bring the x terms to one side of the equation by subtracting x from both sides

$$2x - 2 = -4$$

Get the x term by itself by adding 2 to both sides

$$2x = -2$$

Solve by dividing both sides by 2

$$x = -1$$

b) Solve the equation.

$$2 + \frac{x + 1}{3} = 4$$

Isolate the fraction by subtracting 2 from both sides

$$\frac{x + 1}{3} = 2$$

Get rid of the fraction by multiplying both sides by the denominator (3)

$$x + 1 = 6$$

Get the x term by itself by subtracting 1 from both sides

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$x = 5$


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Unknown on Both Sides


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How do I solve a linear equation with the unknown variable, x, on both sides?

- If a linear equation contains the unknown variable, usually x on both sides start by collecting these terms together on one side of the equation
  - Moving the x term with the smallest coefficient (number in front of x) is easiest
- For example, to solve the equation $4x - 7 = 11 + x$
  - STEP 1
    Move the x term with the smallest coefficient of x
    - The coefficients are 4 and 1 so move the x-term on the right hand side

$$4x - 7 = 11 + x$$

$$(-x) \qquad\qquad (-x)$$

$$3x - 7 = 11$$

  - STEP 2
    Solve the linear equation using the method above

$$3x - 7 = 11$$
$$3x = 18$$
$$x = 6$$

Examiner Tip

- If you have time in the exam, you should substitute your answer back into the equation to check you got it right

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Worked example

Solve the equation

$$4 - 5x = 6x - 29$$

$5x$ is smaller than $6x$ so it's easier to move this term first
Add $5x$ to both sides

$$4 = 6x - 29 + 5x$$

Simplify the right-hand side by adding the $6x$ and the $5x$

$$4 = 11x - 29$$

This could also have been written as $4 = -29 + 11x$
Leave the x's on the right-hand side (avoid 'reflecting' the equation)
Get $11x$ on its own (by adding 29 to both sides)

$$4 + 29 = 11x$$

Work out $4 + 29$

$$33 = 11x$$

Get x on its own (by dividing both sides by 11)

$$\frac{33}{11} = x$$

Work out $33 \div 11$

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$$3 = x$$

The answer currently has x on the right-hand side
At this point, you may reflect the equation to give your answer as $x = ...$

$$x = 3$$


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Forming Linear Equations

Your notes

Before solving an equation you may need to form it from the information given in the question.

How do I form an expression?

- An **expression** is an algebraic statement **without** an equals sign e.g. $3x + 7$
- Sometimes we need to form expressions to help us express unknown values
- If a value is unknown you can represent it by a letter such as x
- You can turn common phrases into expressions
  - Here you can represent the "something" by any letter

2 less than "something"	$x - 2$
Double the amount of "something"	$2x$
5 lots of "something"	$5x$
3 more than "something"	$x + 3$
Half the amount of "something"	$\frac{1}{2}x$ or $\frac{x}{2}$

- You might need to use brackets to show the correct order
  - "something" add 1 then multiplied by 3
    - $(x + 1) \times 3$ which simplifies to $3(x + 1)$
  - "something" multiplied by 3 then add 1
    - $(x \times 3) + 1$ which simplifies to $3x + 1$
- To make the expression as easy as possible choose the smallest value to be represented by a letter
  - If Adam is 10 years **younger** than Barry, then Barry is 10 years **older** than Adam
    - Represent Adam's age as x then Barry's age is $x + 10$
    - This makes the algebra easier, rather than calling Barry's age x and Adam's age $x - 10$
  - If Adam's age is half of Barry's age, then Barry's age is double Adam's age
    - So if Adam's age is x then Barry's age is $2x$
    - This makes the algebra easier, rather than using x for Barry's age and $\frac{1}{2}x$ for Adams's age

How do I form an equation?

- An **equation** is simply an expression with an **equals** sign that can then be solved
- You will first need to form an expression and make it equal to a value or another expression
- It is useful to know alternative words for basic operations:
  - For **addition**: sum, total, more than, increase, etc
  - For **subtraction**: difference, less than, decrease, etc
  - For **multiplication**: product, lots of, times as many, double, triple etc

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- For **division**: shared, split, grouped, halved, quartered etc
- Using the first example above
  - If Adam is 10 years younger than Barry and the sum of their ages is 25, you can find out how old each one is
    - Represent Adam's age as x then Barry's age is $x + 10$
    - We can solve the equation $x + x + 10 = 25$ or $2x + 10 = 25$
- Sometimes you might have **two** unrelated unknown values (x and y) and have to use the given information to form two **simultaneous** equations


Your notes

Worked example

At a theatre the price of a child's ticket is $£x$ and the price of an adult's ticket is $£y$.

Write equations to represent the following statements:

(a) An adult's ticket is double the price of a child's ticket.
(b) A child's ticket is £7 cheaper than an adult's ticket.
(c) The total cost of 3 children's tickets and 2 adults' tickets is £45.

(a) Adult = 2 × Child

$$y = 2x$$

Equivalently you could put $x = \frac{1}{2}y$

(b) Rewrite as:

Adult = Child + £7

$$y = x + 7$$

Equivalently you could put $x = y - 7$ or $y - x = 7$

(c) Total means add

3 × Child + 2 × Adult = £45

$$3x + 2y = 45$$

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Equations & Problem Solving

Your notes

Forming Equations from Shapes

Many questions involve having to form and solve equations from information given about things relating to shapes, like lengths or angles.

How do I form an equation involving the area or perimeter of a 2D shape?

- Read the question carefully to decide if it involves area, perimeter or angles
- If no diagram is given it is almost always a good idea to quickly sketch one
- Add any information given in the question to the diagram
  - This information will normally involve expressions in terms of one or two variables
- If the question involves **perimeter**, figure out which sides are equal length and add these together
  - Consider the properties of the given shape to decide which sides will have equal lengths
    - In a square or rhombus, all four sides are equal
    - In a rectangle or parallelogram, opposite sides are equal
    - If a triangle is given, are any of the sides equal length?
- If the question involves **area**, write down the necessary **formula** for the area of that shape
  - If it is an uncommon shape you may need to **split it up** into two or more common shapes that you can work out areas for
  - In the case you will have to split the length and width up accordingly
- Remember that a **regular** polygon means all the **sides are equal** length
  - For example, a regular pentagon with side length $2x - 1$ has 5 equal sides so its perimeter is $5(2x - 1)$
- If one of the shapes is a circle or part of a circle, use π throughout rather than multiplying by it and ending up with long decimals

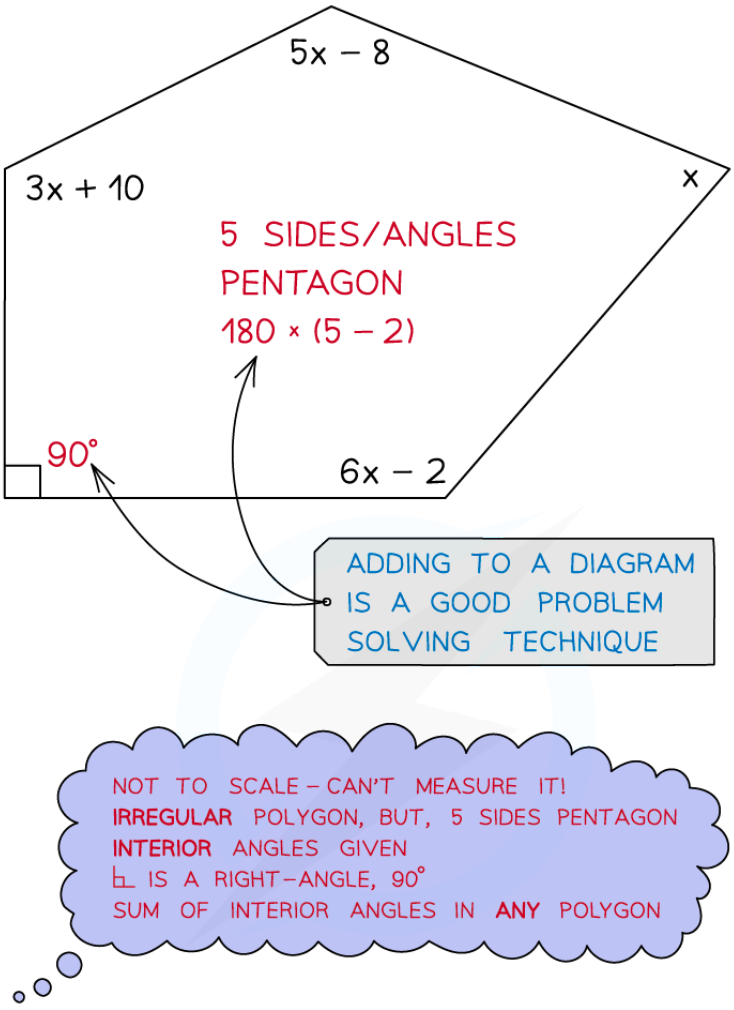
How do I form an equation involving angles in a 2D shape?

- If no diagram is given it is almost always a good idea to quickly sketch one
- Add any information given in the question to the diagram
  - This information will normally involve expressions in terms of one or two variables
- Consider the properties of angles within the given shape to decide which sides will have equal lengths
  - If a triangle is given, how many of the angles are equal?
    - An isosceles triangle has two equal angles
    - An equilateral triangle has three equal angles
  - Consider **angles in parallel lines** (alternative, corresponding, co-interior)
  - In a **parallelogram** or **rhombus**, opposite angles are equal and all four sum to 360°
  - A **kite** has one equal pair of opposite angles
- If the question involves angles, use the formula for the **sum of the interior angles** of a polygon
  - For a polygon of n sides, the sum of the angles will be $180° \times (n - 2)$
  - Remember that a regular polygon means all the angles are equal
- If a question involves an **irregular** polygon, assume all the **angles are different** unless told otherwise
- Look out for key information that can give more information about the angles
  - For example, a trapezium "with a line of symmetry" will have two pairs of equal angles

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e.g. THE DIAGRAM BELOW SHOWS AN IRREGULAR POLYGON. ALL ANGLES ARE IN DEGREES. FIND THE VALUE OF x.



$$\text{SUM OF ANGLES} = 180(n-2)$$
$$= 180(5-2)$$
$$= 180 \times 3$$
$$= 540$$

$$3x + 10 + 5x - 8 + x + 6x - 2 + 90 = 540$$

$$15x + 90 = 540$$

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How do I form an equation involving the surface area or volume of a 3D shape?

- Read the question carefully to decide if it involves surface area or volume
  - Mixing these up is a common mistake
- If no diagram is given it is almost always a good idea to quickly sketch one
- Add any information given in the question to the diagram
  - This information will normally involve expressions in terms of one or two variables
- Consider the properties of the given shape to decide which sides will have equal lengths
  - In a cube all sides are equal
  - All **prisms** have the same shape (**cross section**) at the front and back
- If the question involves volume, write down the necessary **formula** for the area of that shape
  - If it is an uncommon shape the exam question will give you the formula that you need
  - Substitute the expressions for the side lengths into the formula
  - Remember to include brackets around any expression that you substitute in
- It the question involves surface area,
  - You may need to add or subtract some expressions
  - Remember to consider any faces that may be hidden in the diagram
  - STEP 1
    Write down the number of faces the shape has and if any are the same
  - STEP 2
    Identify the 2D shape of each face and write down the formula for the area of each one
  - STEP 3
    Substitute the given expressions into the formula for each one, being careful to identify the correct expression for the dimension
  - STEP 4
    Add the expressions together, double checking that you have one for each of the faces

Examiner Tip

- Use pencil to annotate the diagrams carefully
- You may find that most of your working for a question is on the diagram itself
- Read the question carefully - don't find the area if it wants the perimeter, don't find the volume if it wants the surface area, etc!

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Worked example

A rectangle has a length of $3x + 1$ cm and a width of $2x - 5$ cm.

Its perimeter is equal to 22 cm.

a) Use the above information to form an equation in terms of x.

The perimeter of a rectangle is 2(length) × 2(width).

$$P = 2(3x + 1) + 2(2x - 5)$$

Expand the brackets.

$$2(3x + 1) + 2(2x - 5) = 6x + 2 + 4x - 10$$

Simplify.

$$6x + 2 + 4x - 10 = 10x - 8$$

Set equal to the value given for the perimeter.

$$10x - 8 = 22$$

This equation can be simplified.

$5x - 4 = 11$

b) Solve the equation from part (a) to find the value of x.

Add 4 to both sides.

$$5x - 4 = 11$$

$$5x = 15$$

Divide both sides by 5.

$x = 3$

Your notes

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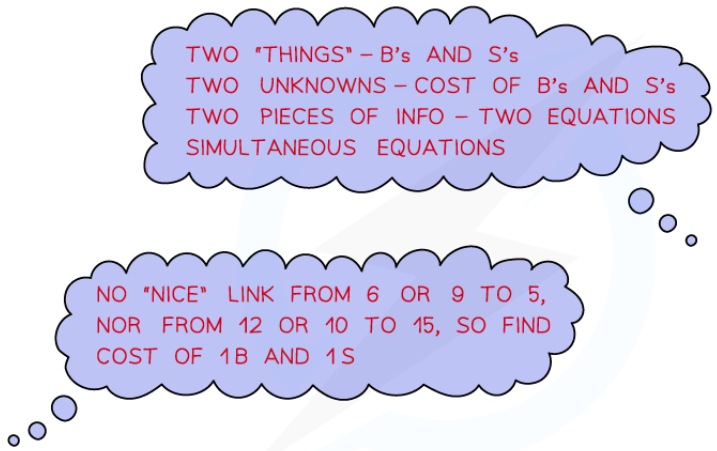
Equations & Problem Solving

Your notes

What is problem solving?

- Problem solving in mathematics involves using several stages, across a variety of topics, to answer a question
- In this set of notes, the problems will involve **equations**
  - These could be **linear equations** or **simultaneous equations**
  - Often, the problem will involve reading the question and setting up an equation, or a set of equations, that can be solved
- With problem solving questions, you never know exactly what's coming, but you can follow a pattern to answer them
- In an ordinary mathematics question you would be given an equation to solve
- In a problem solving question you would have to **generate** the equation ...
  - ... using information from the question
  - ... using your knowledge of standard mathematical results

e.g. AT A FIREWORKS STALL A CUSTOMER PAYS £9 FOR SIX BANGERS AND TWELVE SPARKLERS. ANOTHER CUSTOMER BUYS NINE BANGERS AND TEN SPARKLERS FOR £12.30. FIND THE COST OF 5 BANGERS AND 15 SPARKLERS.



NOTICE HOW MUCH WORK AND THINKING SHOULD HAPPEN IN YOUR HEAD BEFORE ATTEMPTING A SOLUTION!

LET B BE THE PRICE OF BANGERS, IN POUNDS.
LET S BE THE PRICE OF SPARKLERS, IN POUNDS.

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$6B + 12S = 9$ ①
$9B + 10S = 12.3$ ②

① × 3: $18B + 36S = 27$ ③
② × 2: $18B + 20S = 24.6$ ④

③ − ④: $16S = 2.4$

$$S = \frac{2.4}{16}$$

$S = 0.15$

SUB. IN ③: $18B + 36 \times 0.15 = 27$

$18B = 21.6$

$$B = \frac{21.6}{18}$$

$B = 1.2$

SO, $5B + 15S = 5 \times 1.2 + 15 \times 0.15$
$= 8.25$

FIVE BANGERS AND FIFTEEN SPARKLERS WILL COST £8.25

A GOOD ALTERNATIVE QUESTION HERE TO TEST INTERPRETATION OF YOUR ANSWER WOULD BE TO FIND THE CHANGE FROM £10. (£1.75!)

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e.g. AN IRONMAN COMPETITION CONSISTS OF A SWIM, A BICYCLE RIDE AND A MARATHON WITH THE DISTANCES OF EACH SPLIT IN THE RATIO $\frac{x}{10} : 2(2x+5) : x$

COMPETITOR'S COVER A TOTAL DISTANCE OF 137.5 MILES.
FIND THE DISTANCE OF EACH COMPONENT OF THE COMPETITION.



$$\frac{x}{10} + 2(2x+5) + x = 137.5$$

$$x + 20(2x+5) + 10x = 1375$$

x10 TO "GET RID OF FRACTIONS" (GROF)

$$x + 40x + 100 + 10x = 1375$$

EXPAND TO "GET RID OF BRACKETS" (GROB)

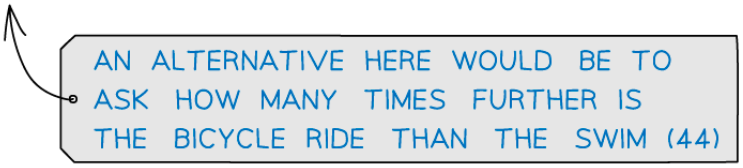
$$51x + 100 = 1375$$
$$51x = 1275$$
$$x = 25$$

SO $\frac{x}{10} : 2(2x+5) : x = \frac{25}{10} : 2(2 \times 25 + 5) : 25$

$= 2.5 : 110 : 25$

SWIM IS 2.5 MILES
BICYCLE RIDE IS 110 MILES
MARATHON IS 25 MILES

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Your notes

- A key feature of problem solving questions is to **interpret** the answer in **context**
- An answer on a calculator may be **1.2**
  - If the question was about money then your final answer should be **£1.20**
- Do **not** necessarily expect whole number (**integer**) or "nice" solutions
- Especially where a calculator is allowed
- **Rounding appropriately** may be one of the skills being tested
  - eg Rounding a value in **cm** only needs to be to one decimal place; so it indicates **mm**
- **Practice**, **experience** and **familiarity** are the keys to solving problems successfully

Examiner Tip

- Do **not** start by focusing on what the question has asked you to find, but on what maths you **can** do
- If your attempt turns out to be unhelpful, that's fine, you may still pick up some marks
- If your attempt is relevant it could nudge you towards the full solution – and full marks!
- Add information to a **diagram** as you work through a problem
  - If there is no diagram, try **sketching** one

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Worked example

A thin horizontal rod is heated at one end. The temperature, T °C, of each point along the rod depends on the distance, x cm, along the rod.

When it is heated from the left-hand side, the temperature is given by

$$T = 100 - 4x$$

When it is heated from the right-hand side, the temperature is given by

$$T = 20 + x$$

One particular point along the rod experiences the same temperature in both situations. What is the temperature at this point?

This is a problem solving question (it's an unfamiliar setup)
You do not know x (how far along the rod this point is) but you know the temperatures are equal at that point
Form an equation by setting the temperatures to be equal

$$100 - 4x = 20 + x$$

Solve this equation to find x

$$\begin{aligned} 100 &= 20 + 5x \\ 100 - 20 &= 5x \\ 80 &= 5x \\ \frac{80}{5} &= x \\ 16 &= x \end{aligned}$$

The point 16 cm along the rod is the one that has 'equal' temperatures
Answer the question by finding the temperature at this point (substitute $x = 16$ back into either equation)

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$$T = 100 - 4 \times 16$$
$$= 100 - 64$$
$$= 36$$

Give your answer in context with units

the temperature at this point is 36°C



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