Cambridge (CIE) IGCSE Maths: Extended

Your notes

Functions

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Introduction to functions

What is a function?

- A **function** is a combination of **one or more mathematical operations** that takes a **set of numbers** and **changes** them into **another set of numbers**
- The numbers being **put into the function** are often called the **inputs**
- The numbers **coming out of the function** are often called the **outputs**
- A function may be thought of as a mathematical "**machine**"
  - For example, for the function "double the number and add 1", the two mathematical operations are "multiply by 2 (×2)" and "add 1 (+1)"
    - Putting 3 in to the function would give $2 \times 3 + 1 = 7$
    - Putting -4 in would give $2 \times (-4) + 1 = -7$
    - Putting x in would give $2x + 1$

What is function notation?

- A function, f, with input x can be written as $f(x) = ...$
  - Letters other than f can be used
    - The letters g, h and j are common but any letter can be used
    - Typically, a **new letter** will be used to define a **new function** in a question
- For example, the function with the rule "triple the number and subtract 4" would be written
  - $f(x) = 3x - 4$
- In such cases, "x" is the **input** and "$f(x)$" is the **output**
- Sometimes functions don't have names like f and are just written as $y = ...$
  - E.g. $y = 3x - 4$

How does a function work?

- A function has an **input** x and an **output** $f(x)$
  - If the input is 2, then the output is $f(2)$
  - If the input is m, then the output is $f(m)$
  - If the input is $t + 5$, then the output is $f(t + 5)$
    - You cannot simplify this output any further

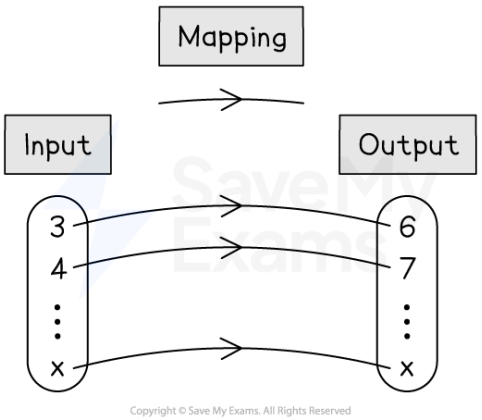
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- If the **function is known**, the **output can be calculated**
  - For example, given the function $f(x) = 2x + 1$
    - $f(3) = 2 \times 3 + 1 = 7$
    - $f(-4) = 2 \times (-4) + 1 = -7$
    - $f(a) = 2a + 1$
- If the **output is known**, an **equation** can be formed and solved to **find the input**
  - For example, given the function $f(x) = 2x + 1$
    - If $f(x) = 15$, then form an equation by replacing $f(x)$ with $2x + 1$
    - $2x + 1 = 15$
    - Solving this equation gives an input of 7
- Note that $f(x) = 15$ and $f(15)$ are very different things:
  - $f(x) = 15$ means an input of x gives an output of 15
  - $f(15)$ means substitute the input 15 into the function

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What is a mapping diagram?

- A **mapping diagram** shows a **set of different inputs** going into the function to become a **set of different outputs**
  - Transforming inputs into outputs is called **mapping**
- For example, a mapping diagram for the function $f(x) = x + 3$ where $x \geq 3$ could be shown as:



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Worked Example

A function is defined as $f(x) = 3x^2 - 2x + 1$.

(a) Find $f(7)$.

Answer:

The input is $x = 7$, so substitute 7 into the expression everywhere you see an x

$$f(7) = 3(7)^2 - 2(7) + 1$$

Calculate

$$\begin{aligned} f(7) &= 3(49) - 14 + 1 \\ &= 147 - 14 + 1 \end{aligned}$$

$$\mathbf{f(7) = 134}$$

(b) Find $f(x+3)$, giving your answer in the form $ax^2 + bx + c$ where a, b and c are integers to be found.

Answer:

The input is $(x+3)$ so substitute $(x+3)$ into the expression everywhere you see an x
This is like replacing x with $(x+3)$

$$f(x+3) = 3(x+3)^2 - 2(x+3) + 1$$

Expand the brackets and simplify
Use that $(x+3)^2 = (x+3)(x+3)$
Be careful with negative signs

$$\begin{aligned} f(x+3) &= 3(x^2 + 6x + 9) - 2(x+3) + 1 \\ &= 3x^2 + 18x + 27 - 2x - 6 + 1 \\ &= 3x^2 + 16x + 22 \end{aligned}$$

$$\mathbf{f(x+3) = 3x^2 + 16x + 22}$$

(a = 3, b = 16, c = 22)

A second function is defined by $g(x) = 3x - 4$.

(c) Find the value of x for which $g(x) = -16$.

Answer:

This is not saying substitute 16 into the function
It says that an input x is substituted into g giving the output -16
To find the input, form an equation by replacing $g(x)$ with $3x - 4$

$$3x - 4 = -16$$

Solve the equation (for example, by adding 4 to both sides, then dividing by 3)

$$\begin{aligned} 3x - 4 &= -16 \\ 3x &= -12 \\ x &= -\frac{12}{3} \end{aligned}$$

$$\mathbf{x = -4}$$

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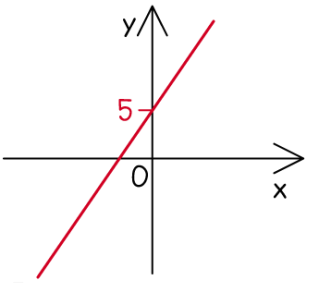
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Domain & range

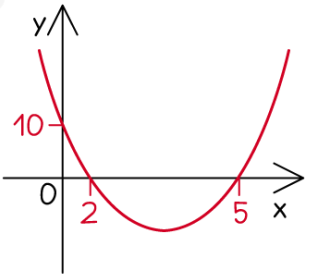
How are functions related to graphs?

- Functions can be represented as graphs on *x* and *y* axes
  - The *x*-axis values are the **inputs**
  - The *y*-axis values are the **outputs**
- To see what graph to plot, replace $f(x) = ...$ with $y = ...$

e.g. f(x) = 2x + 5 is a linear function.
The graph of y = f(x) is a straight line.



e.g. $g(x) = x^2 - 7x + 10$ is a quadratic function.
The graph of y = g(x) is a parabola (quadratic).



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What is the domain?

- The **domain** of a function is the **set of all inputs** that the function is **allowed** to take
- **Domains** can be described in **words**
  - Domains must refer to ***x***
    - not *y* or f(*x*)
  - You can use "**not equal to**" ≠ if needed
  - You can use **inequality signs** if needed

What are examples of domains?

- Examples of domains are below:

- $f(x) = \frac{1}{x}$ takes any x value except 0 (you cannot divide by 0)
  - The domain is "**all values of x except 0**", or simply "**$x \neq 0$**"
- $f(x) = \sqrt{x}$ takes any x value that is not negative (you cannot take the square root of a negative)
  - The domain is "**$x \geq 0$**"
- $f(x) = x^2$ takes any x value (negative x values are fine as inputs)
  - The domain is "**all values of x**"
- $f(x) = 3x + 2$ takes any x value
  - The domain is "**all values of x**"

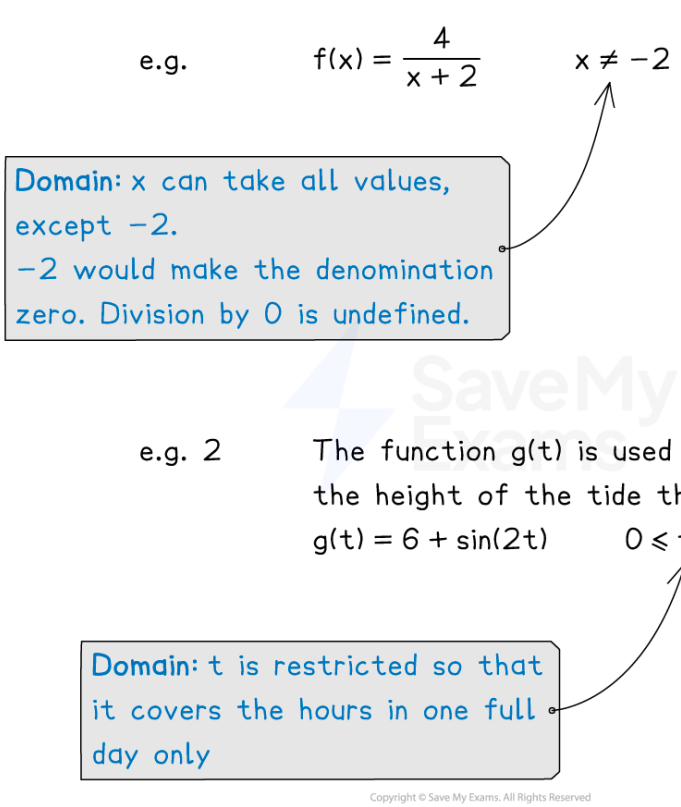
What are restricted domains and values excluded from domains?

- Some domains are **restricted** by choice
  - $f(x) = 3x + 2$ with the domain $0 < x < 5$
    - This question wants to concentrate on that domain only (even though bigger domains exist)
- Some domains must **exclude** certain values (or sets of values)
  - $f(x) = \frac{1}{(x-1)(x+7)}$ must exclude $x = 1$ and $x = -7$ from any domain
    - These two inputs make the function **undefined** (dividing by zero)
  - $f(x) = \sqrt{x-3}$ must exclude $x < 3$ from any domain
    - Any input in $x < 3$ leads to **square-rooting a negative**



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What is the range?

- The **range** of a function is the **set of all outputs** that the function gives out
- **Ranges** can be described in **words**
  - Ranges must refer to **f(x)**
    - not *x* or *y*
  - You can use "**not equal to**" ≠ if needed
  - You can use **inequality signs** if needed
- Ranges **depend on domains**
- Examples of ranges are below:
  - $f(x) = 3x + 2$ with the domain $x > 0$
    - If $x = 0$ then $f(0) = 3(0) + 2 = 2$
    - The range is "$f(x) > 2$"
    - This is because if all inputs are greater than 0, all outputs will be greater than 2
    - This could be seen from a **sketch** or by substituting inputs of $x > 0$ into f(x)
  - $f(x) = x^2$ with domain "all values of *x*"
    - The range is $f(x) \geq 0$

- This is because all values of x get squared (so no negative outputs are created)
- Any negative value that goes in comes out positive
- (0 goes in and comes out as $0^2 = 0$)

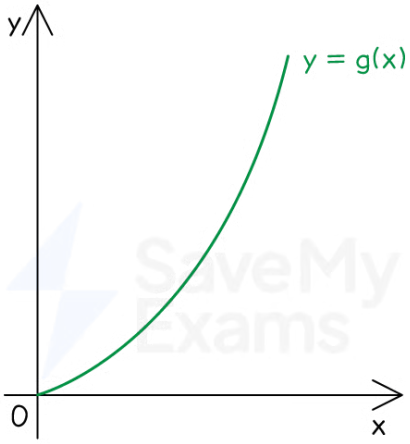


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How can I use graphs to find ranges?

- Ranges are easier if you know the **shapes** of different **types of graphs**
  - For example, the shapes of $y = \frac{1}{x}$, $y = x^2$, $y = x^3$, **trig graphs**, etc
  - They may also involve **graph transformations**

e.g. Sketch the graph of $y = g(x)$ where $g(x) = 3x^2$ $x \geqslant 0$



This will be a transformation of the graph $y = x^2$.
With the domain $x \geqslant 0$ only part of the full graph is required.

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e.g. $f(x) = 2\sin x \quad 0^\circ \leqslant x \leqslant 360^\circ$

a) Explain why the domain of f(x) is restricted to $0^\circ \leqslant x \leqslant 360^\circ$

a) The graph of sinx goes on forever – in both the positive and negative directions.
Restricting the domain to $0^\circ \leqslant x \leqslant 360^\circ$ means the function has only full cycle of sinx only.

b) Write down the range of f(x).

b) $-2 \leqslant f(x) \leqslant 2$

sinx covers values from −1 to 1.
f(x) = 2sinx so these limits are doubled.

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Examiner Tips and Tricks

- Sketching a function in an exam can help to "see" both the domain and range of that function

Worked Example

Two functions are given by

$$f(x) = 10 - x \qquad g(x) = \frac{1}{2x - 1}$$

(a) If the domain of the function f is $2 < x \leq 4$, find the range.

Answer:

The domain is the set of inputs

Substitute $x = 2$ into $f(x)$ to find its output

$$f(2) = 10 - 2$$
$$= 8$$

Substitute $x = 4$ into f(x) to find its output

$$f(4) = 10 - 4$$
$$= 6$$

Think of f(x) = 10 - x as a graph

the graph of $y = 10 - x$

This straight-line graph has a negative gradient
Between $x = 2$ and $x = 4$ the graph decreases from a height of 8 to a height of 6

Relate this to outputs

all outputs are between 6 and 8

Write down the range using f(x)

Remember that the inequality is "equal to" at x = 4, f(x) = 6
(this is the opposite order of "equal to" in the domain)

The range is $6 \leq f(x) < 8$

(b) Write down the value of x that must be excluded from the domain of function g.

Answer:

An input cannot cause the function to divide by zero
Find out when "dividing by zero" would happen

$$2x - 1 = 0$$

Solve to find this value of x (the one that must be excluded)

$$2x = 1$$
$$x = \frac{1}{2}$$

$x = \frac{1}{2}$ must be excluded from the domain

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Composite functions

What is a composite function?

- A **composite function** is a function applied to the **output** of another function
  - The input goes through the 1st function to become an output
  - This output goes through the 2nd function to become a new output

What notation is used for composite functions?

- If $f(x)$ and $g(x)$ are two functions, then
  - $g(f(x))$ is a **composite** function
  - It means the input x goes through **function f first**
    - This gives the output $f(x)$
    - Then this output, $f(x)$, becomes the **input** of function g, giving $g(f(x))$
  - $gf(x)$ is the shorthand **notation used** for $g(f(x))$
    - It means do f first, then g
    - The **order** of applying the functions goes from **right to left**
    - (the letter nearest the bracket goes first)
    - This is often the opposite of what people expect!
  - $fg(x)$ means do $g(x)$ **first** then $f(x)$ **second**
  - $ff(x)$ means apply $f(x)$ twice!
    - This can be written $f^2(x)$
    - This does not mean the same as $[f(x)]^2$

Examiner Tips and Tricks

A good trick in the exam is to write brackets around $gf(x)$ to make it $g(f(x))$, to see that it is "g" of "f(x)".

How do I substitute numbers into composite functions?

- If you are putting a **number** into a composite function

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- put the number into the function closest to (x)
- then make the **output** of the **first** function the **input** of the **second** function

- For example, if $f(x) = 2x + 1$ and $g(x) = \frac{1}{x}$
  - to find $gf(2)$:
    - Put the 2 in as the input of f first
    - $f(2) = 2(2) + 1 = 5$
    - Then put 5 in as the input of g
    - So $gf(2) = g(f(2)) = g(5) = \frac{1}{5}$
  - to find $fg(2)$:
    - Put the 2 in as the input of g first
    - $g(2) = \frac{1}{2}$
    - Then put $\frac{1}{2}$ in as the input of f
    - So $fg(2) = f(g(2)) = f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 2$
  - to find $ff(2)$:
    - $f(2) = 2 \times 2 + 1 = 5$
    - $f(5) = 2 \times 5 + 1 = 11$
    - so $ff(2) = 11$

How do I find composite functions algebraically?

- If you are using algebra, **substitute** the whole **algebraic expression** as your input
  - For example, if $f(x) = 2x + 1$ and $g(x) = \frac{1}{x}$
    - $fg(x) = f(g(x)) = f\left(\frac{1}{x}\right) = 2 \times \left(\frac{1}{x}\right) + 1 = \frac{2}{x} + 1$
    - $gf(x) = g(f(x)) = g(2x + 1) = \frac{1}{2x + 1}$
    - $ff(x) = f(f(x)) = f(2x + 1) = 2(2x + 1) + 1$ which simplifies to $ff(x) = 4x + 3$

Worked Example

Your notes

In this question, $f(x) = 2x - 1$ and $g(x) = (x + 2)^2$.

(a) Find $fg(4)$.

Answer:

"g" is on the inside of the composite function, so apply g first

$$g(4) = (4 + 2)^2 = 6^2 = 36$$

Now apply the function "f" to 36

$$f(36) = 2(36) - 1$$
$$= 72 - 1$$

$$\mathbf{fg(4) = 71}$$

(b) Find $gf(x)$.

Answer:

"f" is on the inside of the composite function so substitute the function f(x) into g(x)
It can help to write $gf(x) = g(f(x))$

$$gf(x) = g(f(x)) = g(2x - 1) = ((2x - 1) + 2)^2$$

Simplify inside the bracket

$$gf(x) = (2x - 1 + 2)^2$$

$$\mathbf{gf(x) = (2x + 1)^2}$$

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Inverse Functions

What is an inverse function?

- An **inverse function** does the **opposite (reverse) operation** of the function it came from
  - E.g. If a function "doubles the number then adds 1"
  - Then its inverse function "subtracts 1, then halves the result"
    - The same inverse operations are used when solving an equation or rearranging a formula
- An inverse function performs the **inverse operations** in the **reverse order**

What notation is used for inverse functions?

- The inverse function of $f(x)$ is written as $f^{-1}(x) = ...$
  - For example, if $f(x) = 2x + 1$
  - The inverse function is $f^{-1}(x) = \frac{x-1}{2}$ or $f^{-1}: x \mapsto \frac{x-1}{2}$
- If $f(a) = b$ then $f^{-1}(b) = a$
  - For example
    - $f(3) = 2 \times 3 + 1 = 7$ (inputting 3 into f gives 7)
    - $f^{-1}(7) = \frac{7-1}{2} = 3$ (inputting 7 into f^{-1} gives back 3)

How do I find an inverse function algebraically?

- The **process** for finding an inverse function is as follows:
  - **Write** the function as $y = ...$
    - E.g. The function $f(x) = 2x + 1$ becomes $y = 2x + 1$
  - **Swap** the xs and ys to get $x = ...$
    - E.g. $x = 2y + 1$
    - The letters change but no terms move
  - **Rearrange** the expression to **make y the subject** again
    - E.g. $x = 2y + 1$ becomes $x - 1 = 2y$ so $y = \frac{x-1}{2}$
  - **Replace y** with $f^{-1}(x) = ...$ (or $f^{-1}: x \mapsto ...$)

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- E.g. $f^{-1}(x) = \frac{x-1}{2}$
  - This is the inverse function
  - y **should not appear** in the final answer

How are inverse functions and composite functions related?

- The **composite function** of f followed by f^{-1} (or the other way round) **cancels out**
  - $ff^{-1}(x) = f^{-1}f(x) = x$
    - If you apply a function to x, then apply its inverse function, you get back x
    - Whatever happened to x gets **undone**
    - f and f^{-1} **cancel each other out** when applied together
- For example, solve $f^{-1}(x) = 5$ where $f(x) = 2^x$
  - Finding the inverse function $f^{-1}(x)$ algebraically in this case is tricky
    - (It is impossible if you haven't studied logarithms!)
  - Instead, you can take f of both sides of $f^{-1}(x) = 5$ and use the fact that ff^{-1} cancel each other out:
    - $ff^{-1}(x) = f(5)$ which cancels to $x = f(5)$ giving $x = 2^5 = 32$

How do I find the domain and range of an inverse function?

- The **domain** of an **inverse function** has exactly the same values as the **range** of the **original function**
  - E.g. If $f(x) = \frac{3}{x+1}$ has a range of $f(x) > 5$
    - then its inverse function, $f^{-1}(x) = \frac{3}{x} - 1$, has the domain $x > 5$
    - Remember to always write domains in terms of x
- The **range** of an **inverse function** has exactly the same values as the **domain** of the **original function**
  - E.g. If $f(x) = \frac{3}{x+1}$ has a domain of $x < -1$
    - then its inverse function, $f^{-1}(x) = \frac{3}{x} - 1$, has the range $f^{-1}(x) < -1$
    - Remember to always write ranges in terms of their function, $f^{-1}(x)$

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Worked Example

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A function $f(x) = 5 - 3x$ has the domain $-2 < x \leq 7$.

(a) Use algebra to find $f^{-1}(x)$.

Answer:

Write the function in the form $y = 5 - 3x$ and then swap the x and y

$$y = 5 - 3x$$
$$x = 5 - 3y$$

Rearrange the expression to make y the subject again

$$x = 5 - 3y$$
$$x + 3y = 5$$
$$3y = 5 - x$$
$$y = \frac{5 - x}{3}$$

Rewrite the answer using inverse function notation

$$f^{-1}(x) = \frac{5 - x}{3}$$

(b) Find the domain of $f^{-1}(x)$.

Answer:

The domain of the inverse function is the range of the original function

Find the range of $f(x)$ by first finding $f(-2)$ and $f(7)$

$$f(-2) = 5 - 3(-2) = 5 + 6 = 11$$
$$f(7) = 5 - 3(7) = 5 - 21 = -16$$

The graph of $y = 5 - 3x$ is a straight line with a negative gradient
Between $x = -2$ and $x = 7$ the graph decreases from a height of 11 to a height of -16

$$\text{The range of } f(x) \text{ is } -16 \leq f(x) < 11$$

Note that the inequality is "equal to" at x = 7, f(x) = -16
(this is the opposite order of "equal to" in the domain)

The domain of $f^{-1}(x)$ takes the same values as range of $f(x)$
Write down the domain of $f^{-1}(x)$
(Remember that domains are always written in terms of x)

$$-16 \leq x < 11$$

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