Mathematics: analysis and approaches
Standard level
Paper 2

Save My Exams Model Answers

1 hour 30 minutes

Instructions to candidates

- A graphic display calculator is required for this paper.
- Section A: answer all questions.
- Section B: answer all questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

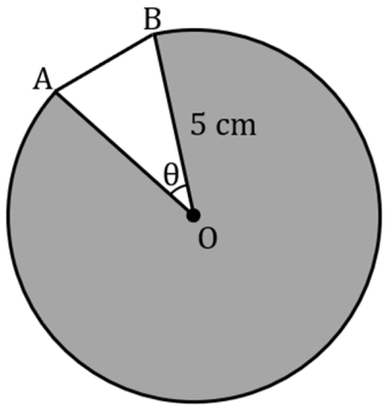
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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

1. [Maximum mark: 6]

A circular pond with radius $0.8\ \text{m}$ contains 16 lily pads. The diagram below shows the shape of each lily pad as part of a circle with centre O and radius $5\ \text{cm}$, $\text{A}\widehat{\text{O}}\text{B} = \theta$.



The lily pads cover $5\ \%$ of the pond's surface.

(a) Find the surface area of each lily pad. [2]

(b) Find the value of θ, giving your answer in radians. [2]

(c) Find the area of the triangle AOB. [2]

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a) area of pond $= \pi r^2 = \pi(0.8)^2 = 0.64\pi \text{ m}^2 = 6400\pi \text{ cm}^2$

5% of that is $0.05 \times 6400\pi = 320\pi \text{ cm}^2$

Use Area of a circle formula from the exam Formula Booklet

So the area of one lily pad is $\frac{320\pi}{16} =$ **$20\pi \text{ cm}^2$** $(= 0.002\pi \text{ m}^2)$

b) The angle for the lily pad's sector of the circle is $(2\pi - \theta)$ radians.

The area of that sector is

Use Area of a sector formula from the exam Formula Booklet

$$\frac{1}{2} r^2 (2\pi - \theta) = \frac{1}{2} (5)^2 (2\pi - \theta) = \frac{25}{2} (2\pi - \theta) \text{ cm}^2$$

So

$$\frac{25}{2} (2\pi - \theta) = 20\pi \implies 2\pi - \theta = \frac{2}{25} \times 20\pi = \frac{8}{5}\pi \implies \theta = 2\pi - \frac{8}{5}\pi$$

$\theta = \frac{2}{5}\pi$ radians $(= \frac{2\pi}{5} = 0.4\pi)$

c) The area of the triangle is

$$\frac{1}{2}(OA)(OB)\sin\theta = \frac{1}{2}(5)(5)\sin\left(\frac{2\pi}{5}\right) = 11.888206\ldots$$

Use Area of a triangle formula from the exam Formula Booklet

11.9 cm^2 (3 s.f.)

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2. [Maximum mark: 6]

A six-sided biased die is weighted in such a way that the probability of obtaining a "one" is $\frac{3}{7}$.

The die is tossed 10 times. Find the probability of obtaining

(a) at most four "ones". [3]

(b) the fourth "one" on the tenth toss. [3]

a) Let X be the number of "ones" rolled in 10 tosses. Then $X \sim B(10, \frac{3}{7})$.

From the GDC, $P(X \leq 4) = P(0 \leq X \leq 4) = 0.56009189$

"at most four"

$P(X \leq 4) = 0.5601$ (4 d.p.)

b) This means getting 3 "ones" in the first 9 tosses, and then getting a "one" on the tenth toss.

Let Y be the number of "ones" rolled in 9 tosses. Then $Y \sim B(9, \frac{3}{7})$.

$P(Y=3) = {}^9C_3 \left(\frac{3}{7}\right)^3 \left(\frac{4}{7}\right)^6 = 0.23020811199...$ You could also get this from the GDC

So the probability of getting the fourth "one" on the tenth toss is:

$P(Y=3) \times \frac{3}{7} = \left({}^9C_3 \left(\frac{3}{7}\right)^3 \left(\frac{4}{7}\right)^6\right) \times \frac{3}{7} = 0.09866062283... =$ 0.0987 (4 d.p.)

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3. [Maximum mark: 5]

A car safety expert is investigating a possible link between the tread depth of a car's tyres and the car's stopping distance.

Using the same car on the same track under the same weather conditions the expert records the average tread depth, $(x \text{ mm})$, from the car's four tyres and the stopping distance, $(y \text{ m})$, when the car's brakes are applied at a particular speed.

Tread depth (x)	6.8	1.4	4.1	0.9	5.7	1.9	3.5	2.6	2.9
Stopping distance (y)	29	45	33.5	49.5	31	42	34	36.5	36

(a) (i) Calculate the Pearson product moment correlation coefficient for these data.

(ii) State the type of linear correlation that is shown between tread depth and stopping distance. [2]

Let L be the regression line of y on x.

(b) (i) Find the equation of L in the form $y = a + bx$.

(ii) Give an interpretation of the values of a and b in the context of the investigation. [3]

a) (i) From the GDC, $r = -0.9177894$

(ii) r is quite close to -1, so there is strong negative correlation between tread depth and stopping distance – the greater the tread depth, the shorter the stopping distance.

b) (i) From the GDC, $a = 47.8925077$ and $b = -3.1722338$

So to 3 s.f., $y = 47.9 - 3.17x$

(ii) $a = 47.9$ means we would expect a car with an average tread depth of 0 mm to have a stopping distance of 47.9 m (i.e., the stopping distance for a car with four 'bald' tires is predicted to be 47.9 m).

$b = -3.17$ means that the stopping distance decreases by 3.17 m for every 1 mm increase in average tread depth.

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4. [Maximum mark: 6]

On 1st January 2021, Nerys invests $\$P$ in an account that pays a nominal annual interest rate of $4.2\,\%$, compounded **monthly**.

The amount of money in Nerys' account **at the end of each year** follows a geometric sequence with common ratio, r.

(a) Find the value of r, giving your answer to four decimal places. [3]

Nerys makes no further deposits to or withdrawals from the account.

(b) Find the year in which the amount of money in Nerys' account will become double the amount she invested. [3]

a) The interest rate for one month is $\frac{4.2\%}{12} = 0.35\%$.

So over one whole year, the balance at the end of the year may be found by multiplying the balance at the beginning of the year by:

$$r = \left(1 + \frac{0.35}{100}\right)^{12} = (1.0035)^{12} = 1.042818\ldots$$

$$r = 1.0428$$

b) We need to find the value of n years for which:

$$\left(1.0035^{12}\right)^{n} \times P = 2P$$

$$\left(1.0035^{12}\right)^{n} = 2$$

$$1.0035^{12n} = 2$$

$$12n = \log_{1.0035}(2)$$

$$a^x = b \iff x = \log_a b$$
$$(a>0,\ b>0,\ a\neq 1)$$
[from the exam Formula Booklet]

$$n = \frac{\log_{1.0035}(2)}{12} = 16.532368\ldots \text{ years}$$

Nerys' money will become double the amount she invested during the year 2037.

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5. [Maximum mark: 6]

Ali makes cone shaped candles which have a radius of $63\ \text{mm}$ and a height of $122\ \text{mm}$.

(a) Find the volume of each candle expressing your answer in the form $a \times 10^k$, $1 \leq a \leq 10$ and $k \in \mathbb{Z}$. [3]

Ali melts three cones down and remoulds them to make one candle in the shape of a sphere.

(b) Find the radius of the sphere, correct to 2 significant figures. [3]

a) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (63)^2 (122) = 161406\pi = 507071.903...$

$V = 5.07 \times 10^5\ \text{mm}^3$ (3 s.f.)

Use Volume of a right cone formula from the exam Formula Booklet

b) 3 cone-shaped candles have a volume of

$3 \times 161406\pi = 484218\pi\ \text{mm}^3$

The volume of a sphere is $V = \frac{4}{3}\pi r^3$.

Use Volume of a sphere formula from the exam Formula Booklet

So

$\frac{4}{3}\pi r^3 = 484218\pi$

$\frac{4}{3} r^3 = 484218$

$r^3 = \frac{3}{4} \times 484218 = 363163.5$

$r = \sqrt[3]{363163.5} = 71.345633...$

$r = 71\ \text{mm}$ (2 s.f.)

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6. [Maximum mark: 7]

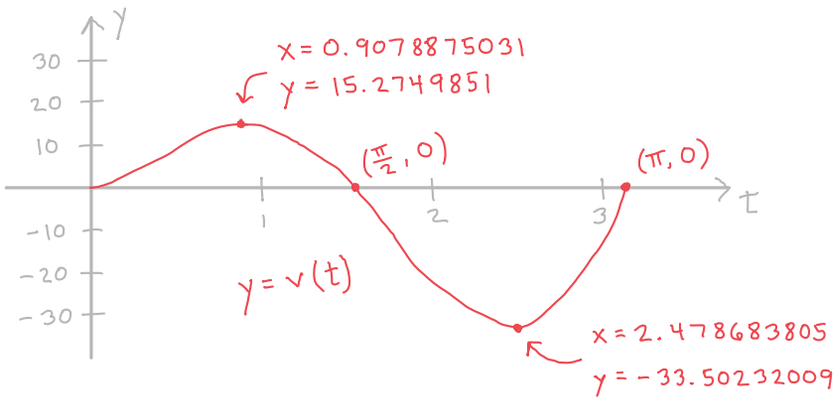
The velocity, v m s^{-1}, of a particle, at time t seconds, is given by $v(t) = 10e^{0.5t} \sin 2t$, $0 \leq t \leq \pi$.

(a) Find the maximum speed of the particle and at what time this occurs. [3]

(b) Find the initial acceleration of the particle. [2]

(c) Show that the distance travelled by the particle is 48.0 m to the nearest 0.1 m. [2]

a) Graphing $y = v(t)$ on the GDC gives:



The speed is greatest when the value of $v(t)$ is furthest from zero (i.e., when $|v(t)|$ has its maximum).

To 3 s.f., the maximum speed is 33.5 ms^{-1}, which occurs at time $t = 2.48$ seconds.

b) Acceleration is the derivative of velocity. Using the GDC to calculate the derivative at time $t = 0$:

$$\frac{d}{dx}\left(10e^{0.5x}\sin 2x\right)\Big|_{x=0} = 20$$

The initial acceleration is 20 ms^{-2}

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c) Distance travelled is the area enclosed by the velocity curve and the time axis on a velocity-time graph.

Using the GDC to calculate the area integral gives:

$$\int_0^{\pi} \left| 10e^{0.5x} \sin 2x \right| dx = 47.98605874$$

Use $A = \int_a^b |y| \, dx$ from the exam Formula Booklet

= 48.0 m to the nearest tenth of a metre

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Section B

7. [Maximum mark: 15]

For cans of a particular brand of soft drink labelled as containing $330\ \text{ml}$, the actual volume, $V\ \text{ml}$, of soft drink in a can is normally distributed with mean 330 and variance σ^2.

The probability that V is greater than 336 is 0.1288.

(a) Find $\text{P}(330 < V < 336)$. [2]

(b) (i) Find σ, the standard deviation of V.

(ii) Hence, find the probability that a can of soft drink selected at random will contain less than $320\ \text{ml}$ of soft drink. [5]

Tilly buys a pack of 24 cans of this soft drink. It may be assumed that those 24 cans represent a random sample. Let L represent the number of cans that contain less than $320\ \text{ml}$ of soft drink.

(c) Find $\text{E}(L)$. [3]

(d) Find the probability that exactly two of the cans contain less than $320\ \text{ml}$ of soft drink. [2]

A can selected at random contains more than $320\ \text{ml}$ of soft drink.

(e) Find the probability that it contains between $330\ \text{ml}$ and $335\ \text{ml}$ of soft drink. [3]

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a) By symmetry of the normal distribution, $P(V > 330) = 0.5$.

So $P(330 < V < 336) = P(V > 330) - P(V > 336) = 0.5 - 0.1288 =$ **0.3712**

b) (i) Using the inverse normal function on the GDC for the standardised normal variable gives:

$$P(Z > z) = 0.1288 \implies z = 1.13208191$$

Then

$$\underbrace{z = \frac{x - \mu}{\sigma}} \implies \sigma = \frac{x - \mu}{z} = \frac{336 - 330}{1.13208191} = 5.299969\ldots$$

This formula is in the exam Formula Booklet

$\sigma = 5.30$ (3 s.f.)

(ii) From the GDC,

$P(V < 320) = 0.0295934014 =$ **0.0296 (4 d.p.)**

c) L is distributed binomially with $n = 24$ and $p = P(V < 320)$.

$$\underbrace{E(L) = np} = 24 \times P(V < 320) = 0.710241\ldots$$

This formula is in the exam Formula Booklet

$E(L) = 0.710$ (3 s.f.)

d) Using the GDC for the binomial distribution gives

$P(L = 2) = 0.12481913 =$ **0.1248 (4 d.p.)**

e) $P(V > 320) = 1 - P(V < 320) = 0.9704065986$

Using the GDC for the normal distribution gives

$P(330 < V < 335) = 0.3272622353$

Use Conditional probability formula from the exam Formula Booklet

So $$P(330 < V < 335 \mid V > 320) = \frac{P(\{330 < V < 335\} \cap \{V > 320\})}{P(V > 320)}$$

$$= \frac{P(330 < V < 335)}{P(V > 320)} = 0.337242\ldots$$

$= 0.3372$ (4 d.p.)

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8. [Maximum mark: 13]

Consider a function f such that $f(x) = 3\sin(2x + \alpha)$ where $0 \leq x \leq 2\pi$ and $0 \leq \alpha \leq \frac{\pi}{2}$.

(a) Write down the amplitude of f. [1]

The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{3}, \frac{3}{2}\right)$.

(b) (i) Find the value of α.

(ii) Find the x-coordinates of the three other points where the graph of $f(x) = \frac{3}{2}$. [4]

The function g is given by $g(x) = pf(x) + q$ where $0 \leq x \leq 2\pi$ and $p, q \in \mathbb{R}$.
The graph of $y = g(x)$ passes through the origin and intercepts the graph of $y = f(x)$ at the points where $f(x)$ is at its maximum value.

(c) Find the values of p and q. [5]

(d) In the case where $\alpha = 0$, show that g can be written in the form $g(x) = k(m\sin x \cos x + n)$, where k, m and n are integers to be found. [3]

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a) **amplitude = 3**

b) (i) $y = f(x)$ passes through $\left(\frac{\pi}{3}, \frac{3}{2}\right)$, therefore

$$f\left(\frac{\pi}{3}\right) = 3\sin\left(2\left(\frac{\pi}{3}\right)+\alpha\right) = 3\sin\left(\alpha+\frac{2\pi}{3}\right) = \frac{3}{2} \implies \sin\left(\alpha+\frac{2\pi}{3}\right) = \frac{1}{2}$$

And $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$, so by symmetry of the sine function possible values of $\alpha+\frac{2\pi}{3}$ are:

$$\frac{\pi}{6},\ \pi-\frac{\pi}{6},\ \frac{\pi}{6}+2\pi,\ \left(\pi-\frac{\pi}{6}\right)+2\pi,\ \frac{\pi}{6}+4\pi,\ \ldots = \frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{13\pi}{6},\ \frac{17\pi}{6},\ \frac{25\pi}{6},\ \ldots$$

But $0 \leq \alpha \leq \frac{\pi}{2}$, so $\alpha+\frac{2\pi}{3} = \frac{5\pi}{6} \implies \alpha = \frac{5\pi}{6}-\frac{2\pi}{3}$ **$\alpha = \frac{\pi}{6}$**

(ii) $0 \leq x \leq 2\pi \implies \frac{\pi}{6} \leq 2x+\frac{\pi}{6} \leq \frac{25\pi}{6}$

And we know from part (i) that

$$3\sin\left(2x+\frac{\pi}{6}\right) = \frac{3}{2} \implies 2x+\frac{\pi}{6} = \frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{13\pi}{6},\ \frac{17\pi}{6},\ \frac{25\pi}{6}$$

Subtracting $\frac{\pi}{6}$ and dividing by 2 gives:

$x = 0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi$

c) $y = g(x)$ goes through the origin, so $g(0) = pf(0)+q = 0$. Therefore:

$$p\left(3\sin\left(2(0)+\frac{\pi}{6}\right)\right)+q = p\left(3\sin\frac{\pi}{6}\right)+q = \frac{3}{2}p+q = 0 \implies q = -\frac{3}{2}p$$

$f(x)$ has a maximum value of 3, and that occurs when

$$2x+\frac{\pi}{6} = \frac{\pi}{2} \text{ or } \frac{5\pi}{2} \implies x = \frac{\pi}{6} \text{ or } \frac{7\pi}{6}$$

$y = g(x)$ intercepts $y = f(x)$ at those maxima, so $g\left(\frac{\pi}{6}\right) = pf\left(\frac{\pi}{6}\right)+q = 3$. Therefore:

$$p\left(3\sin\left(2\left(\frac{\pi}{6}\right)+\frac{\pi}{6}\right)\right)+q = p\left(3\sin\frac{\pi}{2}\right)+q = 3p+q = 3 \implies q = 3-3p$$

Combining $q = -\frac{3}{2}p$ and $q = 3-3p$ gives:

$$-\frac{3}{2}p = 3-3p \implies \frac{3}{2}p = 3 \implies \mathbf{p = 2}$$

Then $q = 3-3p = 3-3(2) = 3-6 \implies$ **$q = -3$**

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d) When $\alpha = 0$, $f(x) = 3\sin 2x$.

Then using the double angle identity for sine* gives

$$f(x) = 3\sin 2x = 3(2\sin x \cos x) = 6\sin x \cos x$$

* This formula is in the exam Formula Booklet

Then

$$g(x) = pf(x) + q = 2(6\sin x \cos x) - 3 = 12\sin x \cos x - 3$$

$$\boxed{g(x) = 3(4\sin x \cos x - 1)}$$

Note: Your GDC could help you with parts of this question.

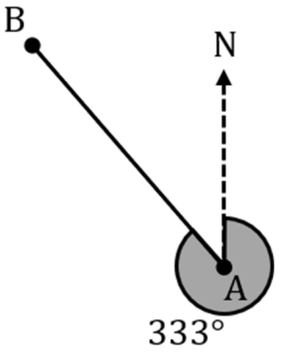
But the entire question could also be answered without any calculator at all, using only familiarity with the sine function and a bit of algebra!

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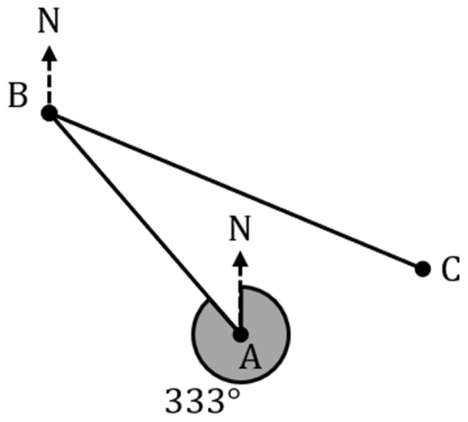
9. [Maximum mark: 16]

Lucy sets out for a bike ride from her house at point A to her friend Heather's house at point B. She rides at an average speed of $7.4\ \text{km/h}$ for 15 minutes, travelling in a straight line on a bearing of $333°$ from her house.

(a) Find the distance from point A to point B. [2]



Lucy and Heather both leave point B on a bearing of $102°$ and continue to ride in that direction for a distance of $2.8\ \text{km}$ until they reach a nature reserve at point C.



At the nature reserve, Lucy gets a puncture so decides to walk back to her house directly from point C. She is able to walk along a straight line the entire way from C to A.

(b) (i) Show that $A\widehat{B}C$ is $51°$.

(ii) Find the distance from the nature reserve to Lucy's house at point A. [5]

(c) Find $B\widehat{A}C$. [3]

(d) Find the bearing that Lucy must take to go home to point A. [3]

It takes Lucy 48 minutes to walk home.

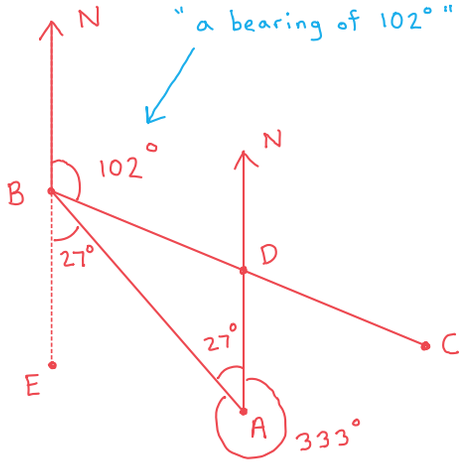
(e) Find the average speed Lucy must have walked on her journey home. [3]

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a) distance = average speed × time, and 15 minutes = 0.25 hours, so

$$AB = 7.4 \times 0.25 = \boxed{1.85 \text{ km}}$$

b) (i)



$\widehat{BAD} = 360 - 333 = 27°$

The two 'north' lines are parallel, so

$\widehat{EBA} = \widehat{BAD} = 27°$

(because they are alternate angles).

Therefore

$\widehat{ABC} = 180 - 102 - 27 = \boxed{51°}$

(ii) By the cosine rule*:

* This formula is in the exam Formula Booklet

$$AC = \sqrt{(AB)^2 + (BC)^2 - 2(AB)(BC)\cos(\widehat{ABC})}$$

$$= \sqrt{1.85^2 + 2.8^2 - 2(1.85)(2.8)\cos(51°)} = 2.177783...$$

$$\boxed{AC = 2.18 \text{ km (3 s.f.)}}$$

c) Using the cosine rule again*:

* This form of the formula is also in the exam Formula Booklet

$$\cos\widehat{BAC} = \frac{(AB)^2 + (AC)^2 - (BC)^2}{2(AB)(AC)} \implies \widehat{BAC} = \cos^{-1}\left(\frac{(AB)^2 + (AC)^2 - (BC)^2}{2(AB)(AC)}\right)$$

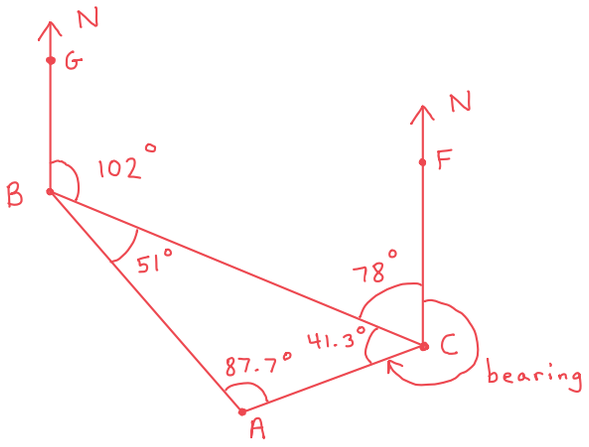
Putting the above values for AB, AC, and BC into that formula gives:

$\widehat{BAC} = 87.686714...$

$$\boxed{\widehat{BAC} = 87.7° \text{ (3 s.f.)}}$$

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d)



$\widehat{ACB} = 180 - 51 - 87.7 = 41.3°$

$\widehat{BCF} = 180 - \widehat{CBG} = 180 - 102 = 78°$

(supplementary angles)

So the bearing is $360 - 78 - 41.3 =$ **240.7° (1 d.p.)**

e) average speed = distance ÷ time, and 48 minutes = 0.8 hours, so

$$\text{average speed} = \frac{AC}{0.8} = \frac{2.177783...}{0.8} = 2.722229...$$

average speed = 2.72 km/h (3 s.f.)

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