Please check the examination details bel	ow before entering your candidate information
Candidate surname	Other names
Centre Number Candidate No Pearson Edexcel International GC	
rearson Edexcer International GC	JL
Time 2 hours	Paper reference 4PM1/01
Further Pure Mat	hematics
Calculators may be used.	Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
 - there may be more space than you need.
- You must NOT write anything on the formulae page.
 Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
 - use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶







International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times \text{slant height}$

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to *n* terms, $S_n = \frac{n}{2} [2a + (n-1)d]$

Geometric series

Sum to *n* terms,
$$S_n = \frac{a(1-r^n)}{(1-r)}$$

Sum to infinity, $S_{\infty} = \frac{a}{1-r} |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$
 for $|x| < 1, n \in \mathbb{Q}$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x} \left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)} \right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{\left[\mathrm{g}(x)\right]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$



Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

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1	Using calculus find the exact value of	3	$\cos 4\theta d\theta$
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Give your answer in the form –	$\frac{\sqrt{a}}{b}$ where a is a prime number and b is an integer.	(4)



(Total for Question 1 is 4 marks)

2	$f(x) = 2x^2 - 12x + 5$
	Given that $f(x)$ can be written in the form $f(x) = a(x+b)^2 + c$ where a, b and c are integers,
	(a) find the value of a , the value of h and the value of c

a j	mu me	value of u	, me varue	e of <i>b</i> and the	value of c	
						(3)

(b)	Hence find the	set of values of x	for which $f(x)$	-37 > 0	
					(4)



3	A geometric series G has first term a and common ratio r	
J		
	The 2nd term of G is $\frac{5}{16}$ and the 5th term of G is $\frac{135}{1024}$	
	(a) Find	
	(i) the value of r	
	(ii) the value of a	(5)
	Given that G is convergent with sum to infinity S	
	(b) find the exact value of S	(2)
		(2)





- 4 (a) On the grid opposite, draw the line with equation
 - (i) y = 2x 4
- (ii) 2x + 3y = 12
- (iii) y + 2x + 2 = 0

(3)

(b) Show, by shading on the grid, the region R defined by the inequalities

$$y \geqslant 2x - 4$$

$$2x + 3y \leq 12$$

$$y + 2x + 2 \geqslant 0$$

(1)

For all points in R, with coordinates (x, y)

$$P = x - 2y$$

(c) find the least value of P

(4)

Question 4 continued 7 6 5 4 3 2 Turn over for a spare grid if you need to redraw your graph.



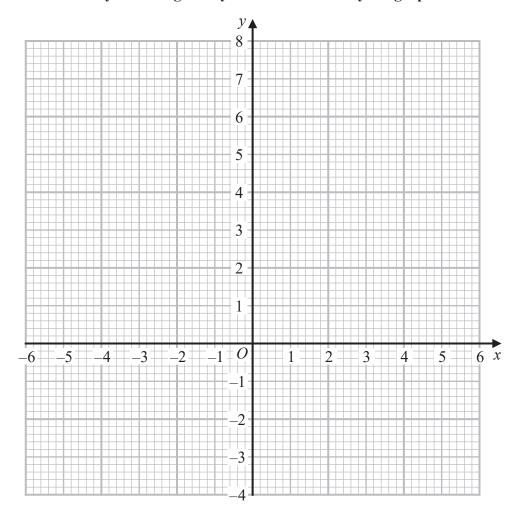
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Question 4 continued	



# **Question 4 continued**

Only use this grid if you need to redraw your graph.





(Total for Question 4 is 8 marks)

5	$f(x) = ax^3 + 5bx^2 + 8ax - 4b$ where a and b are integers	
	Given that	
	(x+2) is a factor of $f(x)$	
	and that	
	when $f(x)$ is divided by $(x + 3)$ the remainder is 21	
	(a) show that $a = 2$ and find the value of $b$	(5)
	(b) Use algebra to solve the equation $f(x) = 0$	(4)





6 In triangle ABC, AC = x cm, AB = (x + 3) cm and  $\angle ABC = 30^{\circ}$ 

Given that  $\angle ACB = \theta^{\circ}$  where  $0 < \theta < 90$ 

(a) show that

(i) 
$$\sin \theta^{\circ} = \frac{x+3}{2x}$$

(ii) 
$$\cos \theta^{\circ} = \frac{\sqrt{3x^2 - 6x - 9}}{2x}$$

(5)

Given that the size of  $\angle BAC$ : the size of  $\angle ABC = 7:2$ 

(b) find the exact value of x

Give your answer in the form  $a + a\sqrt{b}$  where a and b are prime numbers.

(5)



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Question 6 continued	



7	(a) Write down the value of $\log_2 16$	(1)
	Given that $4 + 2\log_4 x = \log_2 y$	
	(b) show that $y = 16x$	(4)
	(c) Hence solve the equation $4 + 2\log_4 x = \log_2(4x + 5)$	
	(1) = 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3	(3)





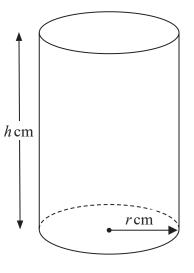


Diagram **NOT** accurately drawn

Figure 1

A solid right circular cylinder has base radius r cm and height h cm as shown in Figure 1.

The cylinder has a volume of  $90\pi \,\mathrm{cm}^3$  and a total surface area of  $S \,\mathrm{cm}^2$ 

(a) Show that 
$$S = 2\pi r^2 + \frac{180\pi}{r}$$

(3)

Given that r can vary,

(b) use calculus to find, to 3 significant figures, the value of r for which S is a minimum, justifying that this value of r gives a minimum value of S

(5)

(c) Find, to 3 significant figures, the minimum value of S

**(2)** 





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Question 8 continued	





(a) Expand  $\frac{1}{\sqrt{1-2x}}$  in ascending powers of x up to and including the term in  $x^3$ Simplify each term as far as possible.

(3)

(b) Show that  $\frac{1}{\sqrt{0.96}} = \frac{5\sqrt{6}}{12}$ 

Show all your working clearly.

**(2)** 

(c) Express  $\frac{1}{5\sqrt{6}-12}$  in the form  $\frac{a\sqrt{6}}{c}+b$  where a, b and c are integers.

Show all your working clearly.

**(2)** 

Using the expansion you found in part (a) with a suitable value of x

(d) find an estimate, to 5 decimal places, of  $\frac{9}{5\sqrt{6}-12}$ 

**(4)** 

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Question 9 continued	



10	The equation	of the	line L	1 1S	y-2x	- 6 =	(
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The point P with coordinates (2, a) lies on  $L_1$ 

(a) Find the value of a

(1)

The line  ${\cal L}_{\scriptscriptstyle 2}$  is perpendicular to  ${\cal L}_{\scriptscriptstyle 1}$  and passes through  ${\cal P}$ 

(b) Show that an equation of  $L_2$  is x + 2y - 22 = 0

**(4)** 

Line  $L_1$  crosses the x-axis at the point A and line  $L_2$  crosses the x-axis at the point B

The point C has coordinates (m, n) such that m > 0 and n < 0

The length of AC is  $5\sqrt{2}$  and the gradient of BC is  $\frac{1}{4}$ 

(c) Find the value of m and the value of n

(9)

(d) Find the area of quadrilateral ACBP

(3)






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Question 10 continued	
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## 11 Given that

$$y = \frac{e^{4x}}{32} \left( 8x^2 - 4x + 1 \right)$$

(a) show that 
$$\frac{dy}{dx} = x^2 e^{4x}$$

**(5)** 

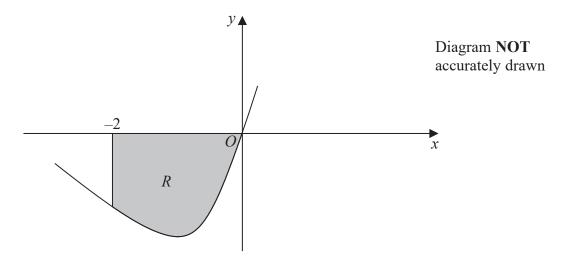


Figure 2

Figure 2 shows part of the curve C with equation  $y = 3xe^{2x}$ 

The finite region R bounded by C, the straight line with equation x = -2 and the x-axis, shown shaded in Figure 2, is rotated though 360° about the x-axis.

(b) Using part (a), find the volume, to 2 significant figures, of the solid formed.

**(4)** 




Question 11 continued			
	(Total for Question 11 is 9 marks)		
	TOTAL FOR PAPER IS 100 MARKS		



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