| Please check the examination d     | etails below before ente | ering your candidate information |
|------------------------------------|--------------------------|----------------------------------|
| Candidate surname                  |                          | Other names                      |
| Pearson Edexcel International GCSE | Centre Number            | Candidate Number                 |
| <b>Thursday 19</b>                 | Novem                    | ber 2020                         |
| Afternoon (Time: 2 hours)          | Paper R                  | eference <b>4PM1/01</b>          |
| Further Pure N Paper 1             | /lathema                 | tics                             |
| Calculators may be used.           |                          | Total Marks                      |

## **Instructions**

- Use **black** ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  - there may be more space than you need.
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

## Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  - use this as a guide as to how much time to spend on each question.

## **Advice**

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶





## **International GCSE in Further Pure Mathematics Formulae sheet**

#### Mensuration

**Surface area of sphere** =  $4\pi r^2$ 

Curved surface area of cone =  $\pi r \times \text{slant height}$ 

Volume of sphere =  $\frac{4}{3}\pi r^3$ 

#### **Series**

#### **Arithmetic series**

Sum to *n* terms,  $S_n = \frac{n}{2} [2a + (n-1)d]$ 

## **Geometric series**

Sum to *n* terms, 
$$S_n = \frac{a(1-r^n)}{(1-r)}$$

Sum to infinity,  $S_{\infty} = \frac{a}{1-r} |r| < 1$ 

#### **Binomial series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$
 for  $|x| < 1, n \in \mathbb{Q}$ 

#### **Calculus**

## **Quotient rule (differentiation)**

$$\frac{\mathrm{d}}{\mathrm{d}x} \left( \frac{\mathrm{f}(x)}{\mathrm{g}(x)} \right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{\left[\mathrm{g}(x)\right]^2}$$

## **Trigonometry**

### Cosine rule

In triangle ABC:  $a^2 = b^2 + c^2 - 2bc \cos A$ 

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

## Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$



# Answer all ELEVEN questions.

Write your answers in the spaces provided.

|   | You must write down all the stages in your working. |      |
|---|---|------|
| 1 | Differentiate with respect to x                     |      |
|   | $6e^{3x^2}\cos 2x$                                  |      |
|   |   | (3)  |
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|   | (Total for Question 1 is 3 ma                       | rks) |



- 2 (a) Using the axes below sketch the line with equation
  - (i) y = 6
- (ii) y + x = 10
- (iii) y = 2x 5

Show the coordinates of any point where each line crosses the coordinate axes.

(3)

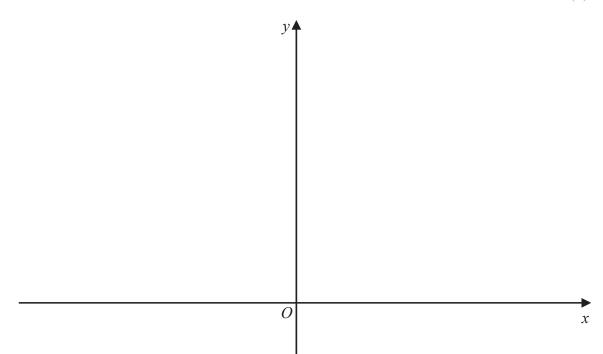
(b) Show, by shading on your sketch, the region R defined by the inequalities

$$y + x \leq 10$$

$$y \geqslant 2x - 5$$

$$x \geqslant 0$$

(1)





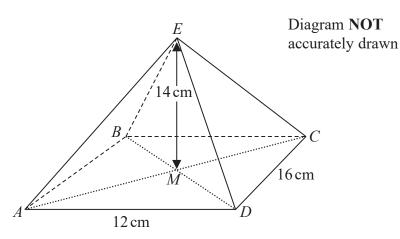


Figure 1

Figure 1 shows the right pyramid ABCDE. The base, ABCD, of the pyramid is a horizontal rectangle with AD = 12 cm and CD = 16 cm. The height ME of the pyramid is 14 cm where M is the point of intersection of the diagonals of the base.

The sloping edges, EA, EB, EC and ED of the pyramid are all of equal length.

| (a) Calculate, to 3 significant figures, the length of a sloping edge. | (3) |
|--|-----|
| Calculate, in degrees to one decimal place, the size of                |     |
| (b) the angle between $AE$ and the base,                               | (2) |
| (c) the angle between the plane <i>AED</i> and the base.               | (3) |
| (c) the angle between the plane AED and the base.                      | (3) |
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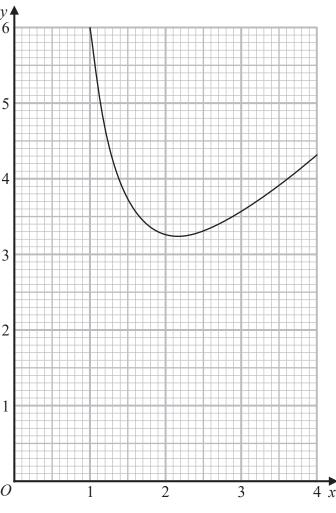


Figure 2

Figure 2 shows the graph of  $y = x + \frac{5}{x^2}$  for  $1 \le x \le 4$  drawn on a grid.

(a) By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, for the roots of the equation

$$x^3 - 4x^2 + 5 = 0$$

in the interval  $1 \leqslant x \leqslant 4$ 

(3)

(b) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, for the root of the equation

$$x^3 - x^2 - 5 = 0$$

in the interval  $1 \leqslant x \leqslant 4$ 

(4)

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| 5 | The points $P$ , $Q$ , $R$ and $S$ have coordinates | rdinates (4, 7) | ), (3, 0), (10, 1) and (11, 8) respectively. |     |
|---|---|-----------------|--|-----|
|   | (a) Show, by calculation, that the l                | ines PR and Q   | QS are perpendicular.                        |     |
|   |   |                 |  | (3) |
|   | (b) Find the exact lengths of (i                    | ) PR            | (ii) QS                                      | (2) |
|   | ( ) F: 14   | 1 0000          |  | (2) |
|   | (c) Find the area of the quadrilater                | al <i>PQRS</i>  |  | (2) |
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| 6 | An arithmetic series $A$ has first term $a$ and common difference $d$ .    |     |
|---|--|-----|
|   | The sum $S_n$ of the first $n$ terms of $A$ is given by $S_n = n(15 + 2n)$ |     |
|   | (a) Find the value of $a$ and the value of $d$ .                           | (4) |
|   | (b) Find the 20th term of A.   | (2) |
|   | Given that $S_{2p} - 2S_p = 1 + S_{(p-1)}$                                 |     |
|   | (c) find the value of p.   | (4) |
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Given that f(x) can be written in the form  $(x + a)^2 + b$ , where a and b are constants,

(2)

- (b) Hence, or otherwise, find
  - (i) the minimum value of f(x)
  - (ii) the value of x for which this minimum occurs.

(2)

The curve C has equation y = f(x)

The line *l* has equation y = x + 5

(c) Use algebra to find the coordinates of the points of intersection of C and l.

**(4)** 

(d) Use algebraic integration to find the exact area of the finite region bounded by C and l.

(5)

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| Question 7 continued |  |
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- 8 Given that  $2xy + 5y = e^x$ 
  - (a) show that  $\frac{dy}{dx} = \frac{y(2x+3)}{2x+5}$

(5)

(b) find the value of  $\frac{dy}{dx}$  when x = 0

(2)

(c) find an equation of the normal to the curve with equation  $2xy + 5y = e^x$  at the point where x = 0

Give your answer in the form px + qy + r = 0 where p, q and r are integers.

**(3)** 



| Question 8 continued |
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BDiagram **NOT**accurately drawn C 2x cm

Figure 3

Figure 3 shows triangle ABC with AB = 12 cm, BC = 6 cm and AC = 2x cm.

The point *D* is the midpoint of AC and BD = 6 cm.

 $\angle ABD = \theta^{\circ}$  and  $\angle DBC = \phi^{\circ}$  where  $\theta \neq 0$  and  $\phi \neq 0$ 

(a) Show that 
$$\cos ADB = \frac{x^2 - 108}{12x}$$

(2)

(b) Hence, or otherwise, show that  $AC = 6\sqrt{6}$  cm.

(4)

(c) Show that  $\sin(\theta^{\circ} + \phi^{\circ}) = \sin \phi^{\circ}$ 

(4)

(d) Hence show that  $\theta = 180 - 2\phi$ 

**(2)** 



| Question 9 continued |  |
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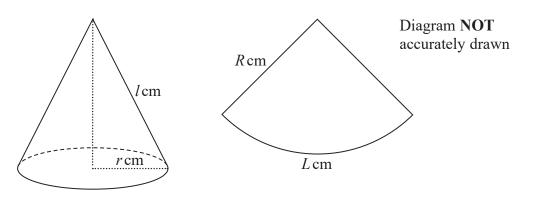


Figure 4

Figure 4 shows a right circular cone with base radius r cm and slant height l cm. Figure 4 also shows a sector of a circle with radius R cm and arc length L cm.

The area of the curved surface of the cone is  $A \text{ cm}^2$ 

By considering how the sector of the circle can be folded to exactly form the curved surface of the cone with R and L suitably chosen,

(a) prove that  $A = \pi r l$ 

**(4)** 

Sand is poured onto a horizontal surface at a constant rate of 1.5 cm<sup>3</sup>/s.

The sand forms a pile in the shape of a right circular cone with its base on the surface. The curved surface area of the cone,  $A \, \text{cm}^2$ , increases in such a way that the height of the cone is always three times the radius of the base of the cone.

Given that  $\frac{dA}{dr} = k\pi r$ , where k is a constant,

(b) find the exact value of k.

(3)

(c) Calculate the rate, in cm<sup>2</sup>/s, to 3 significant figures, at which the curved surface area of the pile is increasing when the height of the pile is 24 cm.









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| Question 10 continued |
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Figure 5

In Figure 5,  $\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$ 

The point C divides OB in the ratio 1:3

The point D is the midpoint of AC

- (a) Find, as a simplified expression in terms of **a** and **b** 
  - (i)  $\overrightarrow{AC}$
- (ii)  $\overrightarrow{OD}$
- (iii)  $\overrightarrow{BD}$

(5)

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The point E is such that  $\overrightarrow{OE} = \lambda \overrightarrow{OA}$ 

Given that E, D and B are collinear

(b) find the value of  $\lambda$ 

(4)

Given that  $\frac{\text{area }\Delta OAC}{\text{area }\Delta OEB} = \mu$ 

(c) find the value of  $\mu$ 

(4)

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