IB · DP · Maths

1 hour 9 questions

Practice Paper 1

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Total Marks /80

1 Let A and B be events such that $\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = 0.75$ and $P(A \cup B) = 0.9$.

Find $P(B|A)$.

(5 marks)

2 Given that $\frac{dy}{dx} = 3x^2 \cos\left(3x^3 + \frac{\pi}{2}\right)$ and that the graph of y passes through the point (0, —1), find an expression for y in terms of x.

(5 marks)

3 (a) The functions f and g are defined such that $f(x) = 6x + 7$ and $g(x) = \frac{x-5}{3}$.

Show that $(f \circ g)(x) = 2x - 3$.

(2 marks)

(b) Given that $(f \circ g)^{-1}(a) = 6$, find the value of a.

(3 marks)

4 (a) i) Expand $(2k-1)^3$.

ii) Hence, or otherwise, show that $(2k-1)^3-(2k-1) = 8k^3-12k^2+4k$.

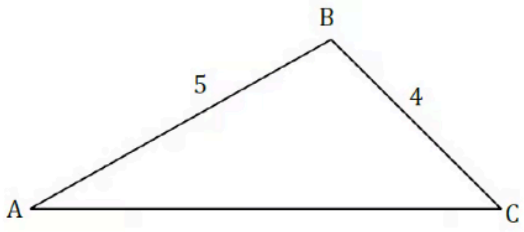
(2 marks)

(b) Thus prove, given $k > 1, k \in \mathbb{N}$, that the difference between an odd natural number greater than 1 and its cube is always even.

(3 marks)

5 (a) The following diagram shows triangle ABC, with $AB = 5$ and $BC = 4$.

diagram not to scale



i) Given that $\sin \widehat{B} = \frac{3}{5}$, find the possible values of $\cos \widehat{B}$.

ii) Given that $\widehat{B}$ is obtuse, find the precise value of $\cos \widehat{B}$.

(3 marks)

(b) Find the length of AC.

(2 marks)

6 (a) Show that $\log_4(\cos 2x + 13) = \log_2 \sqrt{\cos 2x + 13}.$

(3 marks)

(b) Hence or otherwise solve $\log_2(3\sqrt{2}\cos x) = \log_4(\cos 2x + 13)$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

(5 marks)

7 (a) Let $f(x) = \frac{1}{3}x^3 - 2x^2 - 21x - 24$.

Find $f'(x)$.

(2 marks)

(b) The graph of f has horizontal tangents at the points where $x = a$ and $x = b$, $a < b$.

Find the value of a and the value of b.

(3 marks)

(c) i) Find $f''(x)$.

ii) Hence show that the graph of f has a local maximum point at $x = a$.

(2 marks)

(d) i) Sketch the graph of $y = f'(x)$.

ii) Hence, use your answer to part (d)(i) to explain why the graph of f has a local minimum point at $x = b$.

(4 marks)

(e) The tangent to the graph of f at $x = a$ and the normal to the graph of f at $x = b$ intersect At the point (p, q).

Find the value of p and the value of q.

(5 marks)

8 (a) Let $f(x) = \frac{In\ px}{qx}$ where $x > 0$, $p, q \in \mathbb{R}^+$.

Show that $f'(x) = \frac{1 - In\ px}{qx^2}$.

(3 marks)

(b) The graph of f has exactly one maximum point A.

Find the x-coordinate of A.

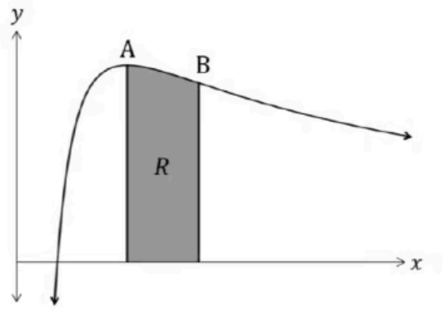
(3 marks)

(c) The second derivative of f is given by $f''(x) = \frac{2\ \text{In}\ px - 3}{qx^3}$. The graph of f has exactly one point of inflexion B.

Show that the x-coordinate of B is $\frac{e^{\frac{3}{2}}}{p}$.

(3 marks)

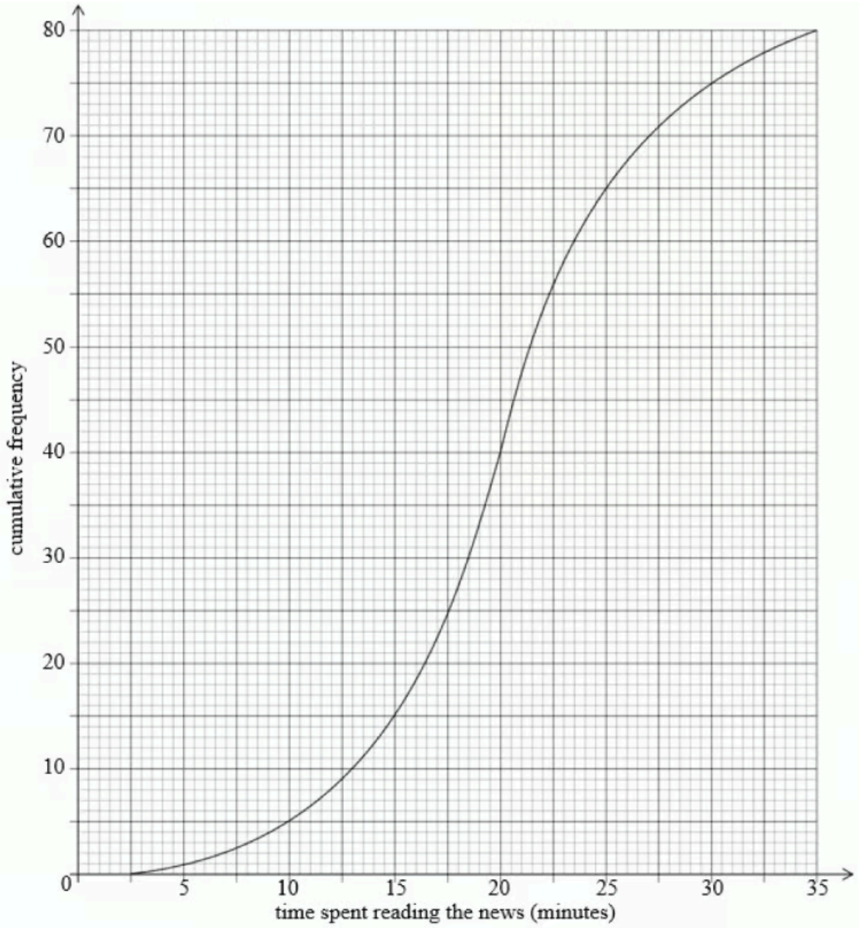
(d) The region R is enclosed by the graph of f , the x-axis, and the vertical lines through the maximum point A and the point of inflexion B.



Calculate the area of R in terms of q and show that the value of the area is independent of p.

(7 marks)

9 (a) A school surveyed 80 of its final year students to find out how much time they spent reading the news on a given day. The results of the survey are shown in the following cumulative frequency diagram.



Find the median number of minutes spent reading the news.

(2 marks)

(b) Find the number of students whose reading time is within 2.5 minutes of the median.

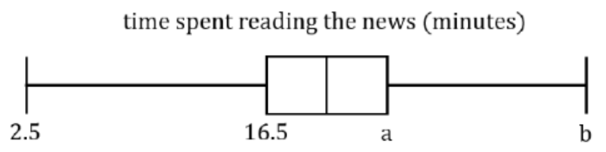
(3 marks)

(c) Only 15% of students spent more than k minutes reading.

Find the value of k.

(3 marks)

(d) The results of the survey can also be displayed on the following box-and-whisker diagram.



Write down the value of b.

(1 mark)

(e) i) Find the value of a.

ii) Hence, find the interquartile range.

(4 marks)

(f) Determine whether someone who spends 30 minutes reading the news would be an outlier.

(2 marks)